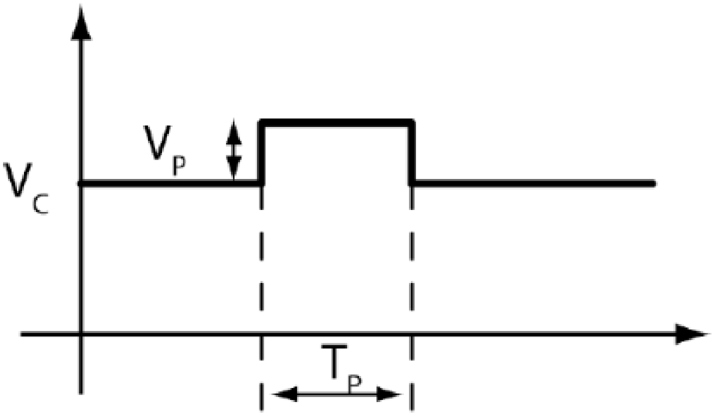
Tutorial – 08



Signal: T_P=50µs variable amplitude V_P repetition rate f_P baseline $V_C \approx$ 10mV	Preamplifier: White noise unilateral spectral density $\sqrt{S_{N,U}}$ = 20nV/$\sqrt{Hz}$ Bandwidth: f_{PA}=1MHz

Rectangular signals coming from a low impedance source are fed to a preamplifier featuring the characteristics reported above. The signals are periodical with repetition rate f_P and the amplitude varies from pulse to pulse. The amplitude V_P of each pulse is to be measured individually.

Considering a repetition rate f_P = 1Hz:

a) if only constant-parameter filters and a peak detector are available, select a filter that allows you to measure the pulse amplitude excluding the baseline and that substantially improves the SNR with respect to that obtainable without any filtering action. Evaluate the minimum amplitude V_P that can be measured in these conditions.

b) Discuss the criteria you would use to select a new filter to further improve the SNR. Propose a filter that can be practically implemented considering that now also switched-parameter filters (e.g. a Gated Integrator) are available. Evaluate the minimum amplitude V_P that can be measured in these conditions.

c) Now consider also a 1/f noise component with f_c=50kHz. Evaluate the impact on the minimum amplitude that can be measured with the two solutions of point a and point b. Discuss the guidelines to design a filter that could allow you to reduce the impact of 1/f noise. Discuss how you could modify the two solutions to improve the sensitivity of your system in these conditions.

Now consider a higher repetition rate f_P = 6kHz:

d) Discuss what you would obtain with the filters used to solve point a and b. Discuss if the proposed filters can be used in these conditions or if they have drawbacks. If none of the two previous solutions can be used in this case, discuss how can you modify at least one of them to make it suitable to be used in these conditions.

A) To remove the baseline $\boldsymbol{V_C}$ we can use a constant parameter high-pass CR filer.
To preserve the shape of the signal, the time constant $\boldsymbol{T_{HP}}$ of the filter must be bigger than $\boldsymbol{T_P}$, furthermore, to preserve the information of the following pulses, $\boldsymbol{T_{HP}}$ must be smaller than the distance between the pulses:

$$T_P \ll T_{HP} \ll \frac{1}{2\pi f_P} \rightarrow T_{HP} = 100T_P \cong 5\ ms \ll \frac{1}{2\pi f_P}$$

To limit the white noise, we can use a constant parameter low-pass RC filter, given we can use a peak-detector, and we are only interested in the amplitude of the signal, we can express the output signal as:

$$y = \frac{V_P}{T_{LP}}\int_0^{\infty} \mathrm{rect}_{T_P}(t) e^{-\frac{t}{T_{LP}}} dt = \frac{V_P}{T_{LP}}\int_0^{T_P} e^{-\frac{t}{T_{LP}}} dt = -V_P\left[e^{-\frac{t}{T_{LP}}}\right]_0^{T_P} = V_P\left[1 - e^{-\frac{T_P}{T_{LP}}}\right]$$

The noise, can instead be written as:

$$\sqrt{\sigma_b^2} = \sqrt{S_V \cdot \frac{\pi}{2} \cdot \frac{1}{2\pi T_{LP}}} = \sqrt{S_V \cdot \frac{1}{4T_{LP}}}$$

Using the optimal value for the time constant $\boldsymbol{T_{LP}} = \frac{4}{5}\boldsymbol{T_P}$, we obtain the minimum measurable amplitude of:

$$SNR = \frac{y}{\sqrt{\sigma_b^2}} = \frac{V_P\left[1 - e^{-\frac{T_P}{T_{LP}}}\right]}{\sqrt{S_V \cdot \frac{1}{4T_{LP}}}} = 1 \rightarrow V_{P,min} = \frac{1}{1 - e^{-\frac{5}{4}}}\sqrt{\frac{S_V}{4T_{LP}}} \cong 2.22\ \mu V$$

B) Keeping the high-pass CR filter, we can replace the low-pass RC filter with a Gated Integrator with $\boldsymbol{T_G = T_P}$, obtaining thus:

$$y = \frac{V_P}{T_P}\int_0^{\infty} \mathrm{rect}_{T_P}(t)\mathrm{rect}_{T_G}(t) dt = \frac{V_P}{T_P}\int_0^{T_P} 1 dt = \frac{V_P}{T_P}[x]_0^{T_P} = V_P$$

The noise is instead:

$$\sqrt{\sigma_b^2} = \sqrt{S_V \cdot \frac{\pi}{2} \cdot \frac{1}{2\pi \frac{T_P}{2}}} = \sqrt{S_V \cdot \frac{1}{2T_P}} \cong 2\ \mu V$$

The minimum measurable amplitude is:

$$SNR = \frac{y}{\sqrt{\sigma_b^2}} = \frac{V_P}{\sqrt{S_V \cdot \frac{1}{2T_P}}} = 1 \rightarrow V_{P,min} = \sqrt{\frac{S_V}{2T_P}} \cong 2\ \mu V$$

C) Considering the presence of the $\boldsymbol{1/f}$ noise, we have, in case A:

$$\sqrt{\sigma_f^2} = \sqrt{S_V \cdot f_C \cdot \ln\left(\frac{f_{LP}}{f_{HP}}\right)} = \sqrt{S_V \cdot \frac{1}{4T_{LP}}} \cong 9.8\ \mu V \rightarrow V_{P,min} = \frac{\sqrt{\sigma_b^2 + \sigma_f^2}}{1 - e^{-\frac{5}{4}}} \cong 14\ \mu V$$

In case B we have instead:

$$\sqrt{\sigma_f^2} = \sqrt{S_V \cdot f_C \cdot \ln\left(\frac{f_{LP}}{f_{HP}}\right)} = \sqrt{S_V \cdot \frac{1}{4T_{LP}}} \cong 10.7\ \mu V \rightarrow V_{P,min} = \sqrt{\sigma_b^2 + \sigma_f^2} \cong 10.9\ \mu V$$

To improve the measurement, we can apply the high-pass filter only to the noise by using a Correlated Double Filtering, choosing to use acquisition windows with a width $\boldsymbol{T_P}$ and distanced $\boldsymbol{T_P}$, we have the weight function:

$$w(\alpha) = \frac{1}{T_P}\mathrm{rect}_{T_P}\left(\alpha - \frac{T_P}{2}\right) - \frac{1}{T_P}\mathrm{rect}_{T_P}\left(\alpha - \frac{3T_P}{2}\right) \rightarrow \begin{cases} f_{LP} \cong \dfrac{1}{2T_P} \\ f_{HP} \cong \dfrac{1}{2\pi T_P} \end{cases}$$

Given the low-pass and high-pass cut off frequencies are of the same order of magnitude, the logarithmic approximation is not accurate, doing the real calculation with the spectrum of the weight function, we obtain:

$$\sqrt{\sigma_n^2} = \sqrt{\sigma_b^2 + \sigma_f^2} = \sqrt{2 \cdot \sigma_{b,GI}^2 + S_V \cdot f_C \cdot 2.3} \cong 7.35\ \mu V \rightarrow V_{P,min} = \sqrt{\sigma_b^2 + \sigma_f^2} \cong 7.35\ \mu V$$