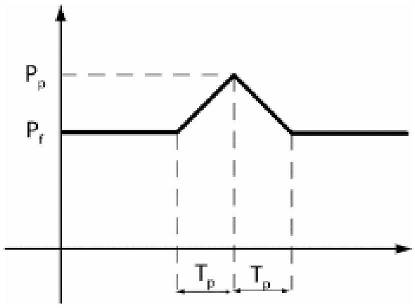
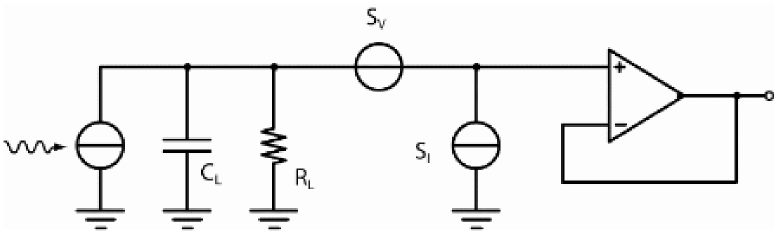
Tutorial – 09

Exam Text of 25/07/2006 (Problem 1)





The signal coming from a photosensor is picked-up by a preamplifier featuring an extremely high input impedance (in the order of 1GΩ), bandwidth limited by a single pole at frequency f_A=100kHz and input-referred wideband noise featuring unilateral spectral density $\sqrt{S_{V,U}}$ = 2 nV/(Hz)$^{1/2}$ e $\sqrt{S_{I,U}}$ = 0,1 pA/(Hz)$^{1/2}$. C_L = 5 pF and R_L = 1 MΩ represent the capacitive and resistive load introduced by the photosensor itself. Before the photosensor an optical filter is present having a narrow optical bandwidth centered around λ=620nm. The photosensor is a phototube featuring a S20 photocathode having quantum efficiency of 5% at 620nm and dark current I_B=1fA. The light pulse reaching the photosensor is shown in figure (left): it has a triangular shape with peak power P_P and duration $2T_P$=1ms superimposed to a continuous background with optical power P_F.

a) Evaluate the minimum optical power that can be measured in absence of background (P_F=0) without using any additional filtering stage.
b) Evaluate the power of the background that would cause an increment by a factor 1.4 of the minimum optical signal that can be measured.
c) Discuss what kind of filtering action is required in order to improve the sensitivity of the system in the conditions of point b); then select a filter and evaluate the minimum optical power that can be measured in these conditions.
d) Discuss and explain the characteristics of the filter that would provide the best SNR; evaluate the corresponding minimum optical power and compare it with the result obtained in point c.

A) Given the sensor output is a current signal, it is better to work directly with current signal, thus we express all noises as current noise, assuming a negligible background we have the following contributes:

Noise of the pre-amplifier (input-referred current and voltage noise:

$$\sqrt{S_{IU}} \cong 100 \frac{fA}{\sqrt{Hz}}$$

$$\sqrt{S_{VU}} \cong 2 \frac{nV}{\sqrt{Hz}} \to \frac{\sqrt{S_{VU}}}{R_L} \cong 2 \frac{fA}{\sqrt{Hz}}$$

Noise introduced by the resistor R_L:

$$\sqrt{S_{IR}} = \sqrt{\frac{4K_bT}{R_L}} \cong 129 \frac{fA}{\sqrt{Hz}}$$

Shot noise due the dark current:

$$\sqrt{S_{IB}} = \sqrt{2q_E I_B} \cong 18 \frac{aA}{\sqrt{Hz}}$$

From their values, we can neglect the input-referred voltage noise of the pre-amplifier and the noise due the dark current, furthermore, we can assume negligible the noise introduced by the signal, the total noise is:

$$S_I = S_{IU} + S_{IR} \to S_V = S_I \cdot \frac{R_L^2}{1 + \omega^2 \tau_L^2}$$

We have an input noise spectrum characterized by a single pole whose time constant is:

$$\tau_L = C_L R_L \cong 5\ \mu s \to f_L = 32\ KHz$$

To simplify the analysis, we can neglect the second pole (the one introduced by the preamplifier), obtaining a conservative estimate of the noise:

$$\sigma_{n,i} = \sqrt{S_I \cdot \frac{\pi}{2} f_P} \cong 36\ pA$$

The minimum current measurable is:

$$I_{P,min} = \sigma_{n,i} \cong 36\ pA$$

The maximum noise associated to the signal (at the peak) is:

$$\sqrt{S_{IB}} = \sqrt{2q_E I_{P,min}} \cong 3.4 \frac{fA}{\sqrt{Hz}}$$

The noise introduced by the signal is negligible, thus the hypothesis made before is valid.

The radiant sensitivity of the detector is:

$$S_D = \eta_D \cdot \frac{\lambda[\mu m]}{1.24} \cong 0.025$$

The minimum optical power measurable is:

$$P_{min} = \frac{I_{P,min}}{S_D} \cong 1.45\ nW$$

B) To increase the minimum optical power of a factor 1.4 the shot noise introduced by the background has to double the total noise power spectrum thus, it must have a value equal to the sum of S_{IU} and S_{IR}:

$$S_{IP} = 2q_e I_P \cong S_{IU} + S_{IR} \to I_P = \frac{S_{IU} + S_{IR}}{2q_e} \cong 83\ nA$$

The optical power needed to generate that background current is:

$$P = \frac{I_P}{S_D} \cong 3.32\ \mu W$$

The total noise spectrum is:

$$S_{tot} = S_{IU} + S_{IR} + S_{IP} = 230 \frac{fA}{\sqrt{Hz}}$$

The effect of the signal shot noise is still negligible:

$$\sqrt{S_{IB}} = \sqrt{2q_E \sqrt{2} I_{P,min}} \cong 4.1 \frac{fA}{\sqrt{Hz}}$$

C) We can use a Gated Integrator with a baseline subtractor (CDF) to sample the signal, for example a zero setting properly sized to avoid noise doubling (long integration windows).
The intrinsic filtering is at $f_L = 32\ KHz$, we can add a low-pass filter at a lower frequency to further limit the noise, for example we can use a Gated Integrator with a bandwidth tailored on the signal.
The bandwidth of the signal is:

$$f_S \cong \frac{1}{T_P} \cong 2\ KHz$$

We can use a gated integrator centred around the peak and with a width T_G, the SNR is equal to:

$$SNR = \frac{\frac{1}{T_G} I_P \int_{-\frac{T_G}{2}}^{\frac{T_G}{2}} \left(1 - \frac{|t|}{T_P}\right) dt}{\sqrt{\frac{S_I}{2T_G}}} = \frac{\frac{2}{T_G} I_P \int_0^{\frac{T_G}{2}} \left(1 - \frac{t}{T_P}\right) dt}{\sqrt{\frac{S_I}{2T_G}}} = \frac{\frac{2}{T_G} I_P \left(\frac{T_G}{2} - \frac{T_G^2}{8T_P}\right)}{\sqrt{\frac{S_I}{2T_G}}} = I_P \left(1 - \frac{T_G}{4T_P}\right) \sqrt{\frac{2T_G}{S_I}}$$

To optimize the Gated Integrator we have to maximize the SNR:

$$\frac{\partial SNR}{\partial T_G} = \frac{I_P}{2}\left(1 - \frac{T_G}{4T_P}\right)\sqrt{\frac{2}{T_G S_I}} - \frac{I_P}{4T_P}\sqrt{\frac{2T_G}{S_I}} = 0 \rightarrow \frac{1}{2T_P} + \frac{1}{4T_P} = \frac{1}{T_G} \rightarrow \frac{3}{4T_P} = \frac{1}{T_G} \rightarrow T_G = \frac{4}{3} T_P$$

Giving us an SNR and a minimum measurable current of:

$$SNR = I_P \left(1 - \frac{T_G}{4T_P}\right)\sqrt{\frac{2T_G}{S_I}} = I_P\left(1 - \frac{1}{3}\right)\sqrt{\frac{8}{3}\frac{T_P}{S_I}} \rightarrow I_{P,min} = \frac{3}{2}\sqrt{\frac{3}{8}\frac{S_I}{T_P}} \cong 9.46\ pA$$

The minimum measurable optical power is:

$$P_{min} = \frac{I_{P,min}}{S_D} \cong 379\ pW$$

D) Let us consider the situation of point B and the baseline removed trough an opportune high pass filter (for example a CDF with a long integration windows to prevent the doubling of the noise).
We can assume the non-stationary noise introduced by the signal is negligible, in this case, the optimum filter is the one with the same shape of the signal, thus:

$$k_{bb}(o) = I_P^2 \int_{-T_P}^{T_P} \left(1 - \frac{|t|}{T_P}\right)^2 dt = 2I_P^2 \int_0^{T_P} 1 - \frac{2t}{T_P} + \frac{t^2}{T_P^2} dt = 2I_P^2 \left[t - \frac{t^2}{T_P} + \frac{t^3}{3T_P^2}\right]_0^{T_P} = 2I_P^2 \frac{T_P}{3}$$

$$SNR = \frac{\sqrt{k_{bb}(o)}}{\sqrt{\frac{S_I}{2}}} = \frac{\sqrt{2I_P^2 \frac{T_P}{3}}}{\sqrt{\frac{S_I}{2}}} = 2I_P\sqrt{\frac{T_P}{3S_I}} \rightarrow I_{P,min} = \sqrt{\frac{3}{4}\frac{S_I}{T_P}} \cong 8.93\ pA$$

The minimum measurable optical power is:

$$P_{min} = \frac{I_{P,min}}{S_D} \cong 357\ pW$$