Tutorial – 04

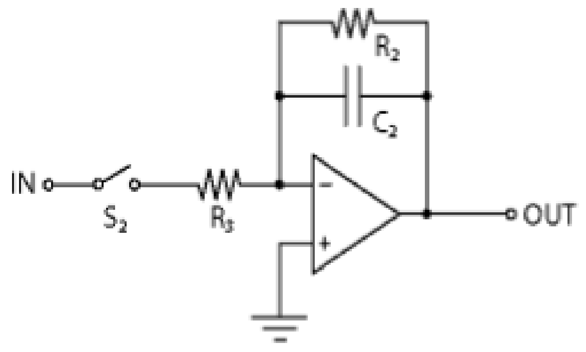

Fig. 1 (a)

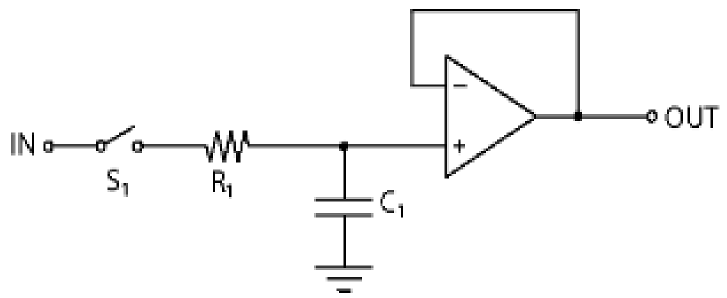

Fig. 1 (b)

Input signal: square pulse with amplitude V_i and duration T_P=10µs.

Noise: wideband (white) component with unilateral spectral density $S_V=(10nV/Hz^{1/2})^2$.

B^2/f^2 component (integrated white noise) with unilateral B=1.6 mV $Hz^{1/2}$.

We want to measure the amplitude of signals coming from a low-impedance source (sensor with preamplifier). An auxiliary synchronism signal is available, which points out the arrival time of the signal. Exploit the two analog filters reported in figure 1. It is possible to use additional linear filters to improve the measurement.

a) Considering only the wideband component of the noise, i.e. overall white noise with unilateral spectral density S_V, select the filter parameters for both filters in order to measure the amplitude of each pulse individually. Evaluate the minimum amplitude that can be measured and compare the result with the optimum filter.

b) At this point, consider also the $1/f^2$ component of the noise. Discuss the weighting function of the matched filter in this case and evaluate the minimum measurable amplitude obtainable with the matched filter. Evaluate the minimum measurable amplitude obtainable with the exact same filters used to solve point a) (**i.e. same parameters**) and compare the results with the minimum signal amplitude that can be measured in these conditions. Discuss if and how the SNR could be improved with a different selection of the filters parameters. Finally, discuss if and how the SNR could be further improved exploiting these filters in conjunction with additional linear filters.

c) The pulse signals are generated by a monitoring system that performs a periodical task. Therefore, the pulses are generated with a repetition rate r_r=1kHz. The signal amplitude slowly varies over time, in a time range of about 1s. Exploit the repetitive nature of the signal to obtain a better SNR and minimum V_i. Explain and discuss how it is possible to exploit the filters of Fig.1 to improve the SNR and explain the differences with respect to the measurement of a single pulse. Select the filter parameters to optimize the measurement and calculate the improvement that can be obtained.

d) Now consider a r_r that slowly varies over time (time range of several seconds) in a range between 1kHz and 2kHz. Discuss if and how it is possible to exploit the same solution of point c) and discuss the results obtainable in this case.

e) Finally consider a statistical repetition rate with a Poisson distribution over time with average repetition rate of 1kHz. Discuss and compare the results that could be achieved with the solution of point b.

A) To maximize the signal acquired we must use big time constants ($T_F \gg T_P$) and close the switch only for T_P, the difference between the ratemeter and the boxcar integrator is that in the ratemeter the output decreases exponentially also when the switch is closed, so, if we wish to achieve the maximum SNR we must sample the output at the end of T_P.
Given the assumptions made before, we can approximate both the weight function as a series of rectangles slowly decaying (with different time constants depending on the ratemeter or the boxcar integrator), that means we can use both the filters to implement a Gated Integrator.
The minimum amplitudes measurable for both filters (considered as Gated Integrator) is:

$$V_{min} = \sqrt{\frac{S_V}{2T_P}} \cong 2.24\ \mu V$$

Given the pulse is rectangular, a GI we implement an optimum filter (or, at least a very good approximation).

B) If we introduce a Flicker noise the assumptions made before to implement the optimum filter are no longer valid, we need to apply a whitening filter and the find the new matched filter.

$$\frac{B^2}{f_{nc}^2} = S_V \rightarrow f_{nc} = \frac{B}{\sqrt{S_V}} \cong 160\ KHz$$

To whiten the noise, we need to cancel the pole in zero and the zero in f_{nc}, to achieve this, we can use a high-pass CR network with a time constant $T_{nc} = \frac{1}{2\pi f_{nc}} \cong 1\ \mu s$.
Using this filter, we can whiten the noise, but the filter affects the signal, it turns the rectangular shape in a positive exponential decay followed by a negative exponential decay after T_P:

$$x(t) = V_P\left[u(t)e^{-\frac{t}{T_{nc}}} - u(t-T_P)e^{-\frac{t-T_P}{T_{nc}}}\right]$$

The matched filter is the filter that implements a weight function with the same shape of the signal, in that case, we get an optimum SNR squared of:

$$E = \int_{-\infty}^{\infty} x^2(t)dt \cong V_P^2 \int_{-\infty}^{\infty} u(t)e^{-\frac{2t}{T_{nc}}} + u(t-T_P)e^{-2\frac{t-T_P}{T_{nc}}}dt = 2V_P^2\int_0^{\infty} e^{-\frac{2t}{T_{nc}}}dt = V_P^2 T_{nc}$$

$$SNR_{opt} = \frac{\sqrt{E}}{\sqrt{\frac{S_V}{2}}} \cong V_P\sqrt{\frac{2T_{nc}}{S_V}} \rightarrow V_{min} = \sqrt{\frac{S_V}{2T_{nc}}} \cong 7.09\ \mu V$$

Considering the filters discussed at point **A** we have an output signal equals to:

$$y = \int_{-\infty}^{\infty} x(t)\mathrm{rect}_{T_G}\left(t-\frac{T_G}{2}\right)dt = V_P\int_0^{T_G} V_P e^{-\frac{t}{T_{nc}}}dt = V_P T_{nc}\left(1-e^{-\frac{T_G}{T_{nc}}}\right) \rightarrow SNR = \frac{V_P T_{nc}\left(1-e^{-\frac{T_G}{T_{nc}}}\right)}{\sqrt{\frac{S_V}{2}T_G}}$$

Using $T_G = T_P$ and a unitary gain, we obtain:

$$V_{P,min} = \frac{1}{1-e^{-10}}\sqrt{\frac{S_V T_P}{2T_{nc}^2}} \cong 22.48\ \mu V$$

If we use the optimal value of $T_G = 1.25 \cdot T_{nc}$ and the same unitary gain, we obtain:

$$V_{P,min} = \frac{1}{1-e^{-\frac{5}{4}}}\sqrt{\frac{5}{8}\frac{S_V}{T_{nc}}} \cong 11.11\ \mu V$$

We can choose to use two integration windows with a $T_G = 1.25 \cdot T_{nc}$, and then subtract the latter from the former, in this case we obtain an improvement of a factor $\sqrt{2}$ with respects to the single GI above:

$$V_{P,min} = \frac{1}{1-e^{-\frac{5}{4}}}\sqrt{\frac{5}{16}\frac{S_V}{T_{nc}}} \cong 7.86\ \mu V$$

C) Using the information about the repetition of the signal, we can improve the SNR by acquiring multiple replicas of the same signal, considering that the signal varies with $\boldsymbol{T_R \cong 1\ s}$, the weight function must be zero after $\boldsymbol{T_R}$. Considering the ratemeter and a sampling frequency $\boldsymbol{f_s = r_r = \frac{1}{T_s}}$, we can choose a time constant of the filter $\boldsymbol{T_F = RC}$ much bigger than the signal duration $\boldsymbol{T_P}$ (to use the GI model), but smaller than $\boldsymbol{T_R}$, given the exponential decay of the weight function we choose $\boldsymbol{T_F = 200\ ms}$, (in $\boldsymbol{5\tau}$ the exponential goes to zero):

$$\boldsymbol{T_F = \frac{T_R}{5} \cong 200\ ms}$$

The improvement factor from multiple samples is proportional to the square root of the number of samples (proportional to the ratio of the duration of the weighting and the duration of each sample):

$$\boldsymbol{IF = \sqrt{2N} = \sqrt{2T_F f_s} = 20 \rightarrow V_{P,min} = \frac{V_{P,min,0}}{IF} \cong 112\ nV}$$

The same process can be applied for the boxcar integrator, in that case the weight function decays only when the switch is closed, so considering a switch closed for a period $\boldsymbol{T_P}$ each $\boldsymbol{T_G}$ we can choose $\boldsymbol{T_B}$ as:

$$\boldsymbol{T_B = \frac{T_R}{5}\frac{T_P}{T_G} \cong 2ms}$$

The improvement factor from multiple samples is proportional to the square root of the number of samples, (proportional to the ratio of the duration of the weighting and the duration of each sample):

$$\boldsymbol{IF = \sqrt{2N} = \sqrt{2\frac{T_B}{T_G}} = 20 \rightarrow V_{P,min} = \frac{V_{P,min,0}}{IF} \cong 112\ nV}$$

D) If the repetition rate $\boldsymbol{r_r = f_s}$ of the signal change the SNR of the boxcar integrator remain the same calculated at point **C** (does not depend on the frequency) for the ratemeter we have instead an improvement factor of:

$$\boldsymbol{IF_{max} = \sqrt{2T_F f_s} \cong 28.3}$$

So, an improvement of a factor $\boldsymbol{\sqrt{2}}$ with respect to the situation before:

$$\boldsymbol{V_{P,min} = \frac{V_{P,min,0}}{IF} \cong 79.2\ nV}$$

E) Considering a statistical, Poisson distributed repetition rate $\boldsymbol{r_r}$ centred around $\boldsymbol{\overline{r_r} = 1\ KHz}$ we obtain the same results that we would have obtained considering a constant repetition rate $\boldsymbol{r_r = \overline{r_r} = 1\ KHz}$, obtaining thus the same results found at point **C**.