Tutorial – 07

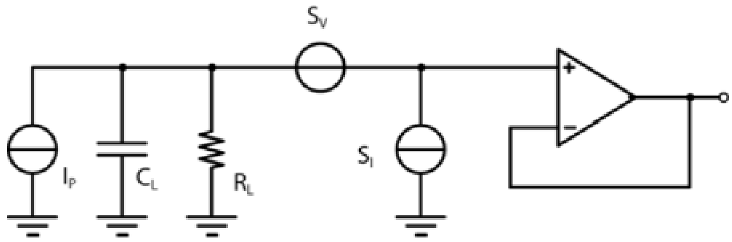

Fig. 1

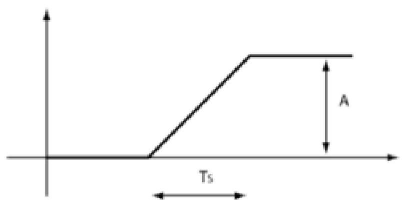

Fig. 2

The signal of a photodetector is read out (Fig. 1) by means of a preamplifier featuring an extremely high input impedance. The bandwidth of the preamplifier is limited by a single pole at frequency f_P=10MHz and the noise referred to the preamp input is the sum of two wideband contributions having **unilateral** spectral densities $\sqrt{S_{V,U}}$ = 1nV/$\sqrt{\text{Hz}}$ and $\sqrt{S_{I,U}}$ = 0.05pA/$\sqrt{\text{Hz}}$. C_L=10pF is the total capacitance between the preamp input and ground and R_L=200MΩ is the total load resistance. The voltage signal on the load has an almost-step shape with amplitude A, with linear rising edge of duration T_S=20µs (Fig. 2). We want to measure the signal amplitude A with good sensitivity, i.e. with a SNR higher than 10 even for a small A.

a) Evaluate the noise at the preamp output and therefore the minimum amplitude A that can be measured without using any additional filter.
b) Discuss the weighting function of the filter that would allow you to obtain the best sensitivity in this system. Evaluate the noise of the measurement that you would have exploiting this filter and the minimum amplitude A that could be measured in these conditions.
c) Select a filter that can be practically implemented and that is a good approximation of the weighting function discussed in the solution of point b). Select the filter parameters, evaluate the noise of the measurement carried out with this filter and evaluate the minimum measurable amplitude in these conditions.
d) Now consider an additional 1/f noise component with a corner frequency f_C=1kHz. Discuss if and how this contribution can affect the sensitivity of the measurement and discuss which additional filters would you use to limit 1/f noise, if needed. A qualitative evaluation is sufficient.

A) The total power spectral density at the input of the preamplifier is given by the contributes of the preamplifier's own noise sources and the noise source of the resistor, the noise spectral density of the resistor is:

$$\sqrt{S_{R,U}} = \sqrt{\frac{4k_BT}{R_L}} \cong 0.1\frac{fA}{\sqrt{Hz}}$$

The noise power spectral density of the resistor is negligible, the output noise can thus be considered as deriving only from the preamplifier (i.e., its current and voltage input referred sources).

We can approximate the transfer function of the input referred source as a single pole response:

$$S_n(\omega) = S_V\left|\frac{1}{1+j\omega\tau_{AMP}}\right|^2 + S_I\left|\frac{R_L}{1+j\omega R_LC_L}\right|^2\left|\frac{1}{1+j\omega\tau_{AMP}}\right|^2 \cong S_V\left|\frac{1}{1+j\omega\tau_{AMP}}\right|^2 + S_I\left|\frac{R_L}{1+j\omega R_LC_L}\right|^2$$

We can express the amplitude of the output noise using the Equivalent Noise BandWidth (ENBW):

$$\sqrt{\sigma_n^2} \cong \sqrt{S_{V,U}\cdot\frac{\pi}{2}f_P + S_{I,U}\cdot R_L^2\cdot\frac{\pi}{2}\cdot\frac{1}{2\pi R_LC_L}} = \sqrt{S_{V,U}\cdot\frac{\pi}{2}f_P + S_{I,U}\cdot R_L^2\cdot\frac{1}{4R_LC_L}} \cong 112\ \mu V$$

To achieve a minimum SNR of **10**, we need a minimum input amplitude of:

$$SNR = \frac{V_P}{\sqrt{\sigma_n^2}} = 10 \rightarrow V_{P,min} \cong 1.12\ mV$$

B) To improve the sensitivity of the system we can add an optimum filter, to do so, we have to design a whitening filter to flatten the noise spectrum (the optimum filter theory is valid only for signals in white noise).

The whitening filter must have a zero to compensate the pole at $f_L = \frac{1}{2\pi R_L C_L} \cong 80\ Hz$ and a pole to flatten the response at the frequency f_C at which the amplitude of the two noise sources is the same:

$$f_C = f_L \cdot R_L \sqrt{\frac{S_I}{S_V}} \cong 800\ KHz \rightarrow H_W(\omega) = \frac{f_L}{f_C} \cdot \frac{1 + j\omega f_L}{1 + j\omega f_C} = 10\,000 \cdot \frac{1 + j\omega f_L}{1 + j\omega f_C}$$

After the whitening filter, we can consider the noise as having a white spectrum:

$$S_{n,U} = S_{V,U}$$

The whitening filter also affects the signal, to find the signal at the output, we can consider the effect of the two filters (the low-pass input filter and the whitening filter) as a single low-pass filter with a cut-off frequency f_C:

$$H(\omega) = H_L(\omega) \cdot H_W(\omega) = \frac{1}{1 + j\omega f_L} \cdot \frac{1 + j\omega f_L}{1 + j\omega f_C} = \frac{1}{1 + j\omega f_C}$$

To proceed with the analysis, we need to find the expression of the original signal I_P.

The signal at the output of the first low-pass filter is a ramp, given the short time of observation ($T_S \cong 20\ \mu s$) and the time constant of the filter ($T_L = R_L C_L \cong 2\ ms$), we have that the original signal is compatible with a rectangular pulse ($T_L \gg T_S$, we are observing the start of the exponential curve):

$$\left(1 - e^{-\frac{t}{T_L}}\right) \cong \frac{t}{T_L}$$

Regarding the amplitude, we have that the voltage at the output is proportional to the charge in the capacitor:

$$A = \frac{x \cdot T_P}{C_L} \rightarrow x = \frac{A \cdot C_L}{T_S}$$

We can thus express the signal I_P at the output of the sensor as:

$$I_P = x \cdot \mathrm{rect}_{T_S}\left(T - \frac{T_S}{2}\right)$$

Having obtained the information's on the original signal, we obtain at the output of the two filters:

$$y = \begin{cases} x \cdot R_L \cdot \left[1 - e^{-\frac{t}{T_C}}\right] = A \cdot \frac{T_L}{T_S} \cdot \left[1 - e^{-\frac{t}{T_C}}\right] & for \quad 0 \le t \le T_S \\ x \cdot R_L \cdot e^{-\frac{t - T_S}{T_S}} = A \cdot \frac{T_L}{T_S} \cdot e^{-\frac{t - T_S}{T_S}} & for \quad T_S \le t \end{cases}$$

The optimum filter is the one whose weight function matches the shape of the signal, in that case, the signal at the output can be expressed as:

$$k_{bb}(0) = \int_{-\infty}^{\infty} b^2(t)dt = \int_0^{T_P} 1 - 2e^{-\frac{t}{T_{SP}}} + e^{-\frac{2t}{T_{SP}}} dt + \int_{T_P}^{\infty} e^{-2\frac{t - T_P}{T_{SP}}} dt \cong T_P - T_{SP}$$

To achieve a minimum SNR of 10, we need a minimum input amplitude of:

$$SNR = x\frac{\sqrt{k_{bb}(0)}}{\sqrt{S_n}} = x\frac{\sqrt{T_P - T_{SP}}}{\sqrt{S_n}} = V_P \cdot \frac{T_L}{T_S} \cdot \frac{\sqrt{T_P - T_{SP}}}{\sqrt{S_n}} = 10 \rightarrow V_{P,min} = 10 \cdot \frac{T_S}{T_L} \cdot \sqrt{\frac{S_n}{T_P - T_{SP}}} \cong 159\ \mu V$$

C) A good approximation of the of the optimum filer analysed before can be realized with a Gated Integrator (with $T_G = T_S$ and a weight $\frac{1}{T_S}$ to simplify the analysis), and $t_m = T_C \cdot \ln(2)$ (as calculated in practice 05) we obtain:

$$k_{wb}(0) = \int_{t_m}^{t_m + T_S} b(t)dt = \int_{t_m}^{T_S} 1 - e^{-\frac{t}{T_C}} dt + \int_{T_S}^{t_m + T_S} e^{-\frac{t - T_S}{T_C}} dt = T_S - T_C \cdot \ln(2)$$

To achieve a minimum SNR of 10, we need a minimum input amplitude of:

$$SNR = x \cdot \frac{\frac{1}{T_S} k_{bb}(0)}{\sqrt{S_n \cdot \frac{1}{2T_S}}} = V_P \cdot \frac{T_L}{T_S} \cdot \frac{T_S - T_C \cdot \ln(2)}{\sqrt{\frac{S_n}{2} \cdot T_S}} = 10 \rightarrow V_{P,min} = 10 \cdot \frac{T_S}{T_L} \cdot \frac{\sqrt{\frac{S_n}{2} \cdot T_P}}{T_S - T_C \cdot \ln(2)} \cong 159.23\ \mu V$$

The result is good enough with respect to the real optimum filter, there isn't much loss.

D) Taking into account the presence of $\mathbf{1/f}$ noise, with a noise corner frequency $\boldsymbol{f_{NC} = 1\ KHz}$, we must consider how the flicker noise affect the SNR.

Considering for example a high-pass CR filter with a time constant $\boldsymbol{\tau_H \cong 100 \cdot T_S \cong 2\ ms}$, and keeping the same whitening filter considered before ($\boldsymbol{f_C = 10\,000 \cdot f_L = \frac{10\,000}{2\pi R_L C_L} \cong 800\ KHz}$), we get:

$$\boldsymbol{f_H = \frac{1}{2\pi\tau_H} \cong 80\ Hz \rightarrow \sqrt{\sigma_f^2} = \sqrt{S_{V,U} \cdot f_{NC} \cdot \ln\left(\frac{f_C}{f_H}\right)} = \sqrt{S_{V,U} \cdot f_{NC} \cdot \ln(10\,000)} \cong 96\ nV}$$

Considering now the effect of the Gated Integrator (with $\boldsymbol{T_G = T_S}$), we have:

$$\boldsymbol{f_H = \frac{1}{2\pi\tau_H} \cong 80\ Hz \rightarrow \sqrt{\sigma_f^2} = \sqrt{S_{V,U} \cdot f_{NC} \cdot \ln\left(\frac{1}{f_H \cdot 2T_S}\right)} \cong 76\ nV}$$

In both cases, the intensity of the flicker noise is negligible with respect to the noise calculated at point **A** by at least $\mathbf{3}$ order of magnitude, we can thus assume that the flicker noise doesn't affect significatively the SNR.