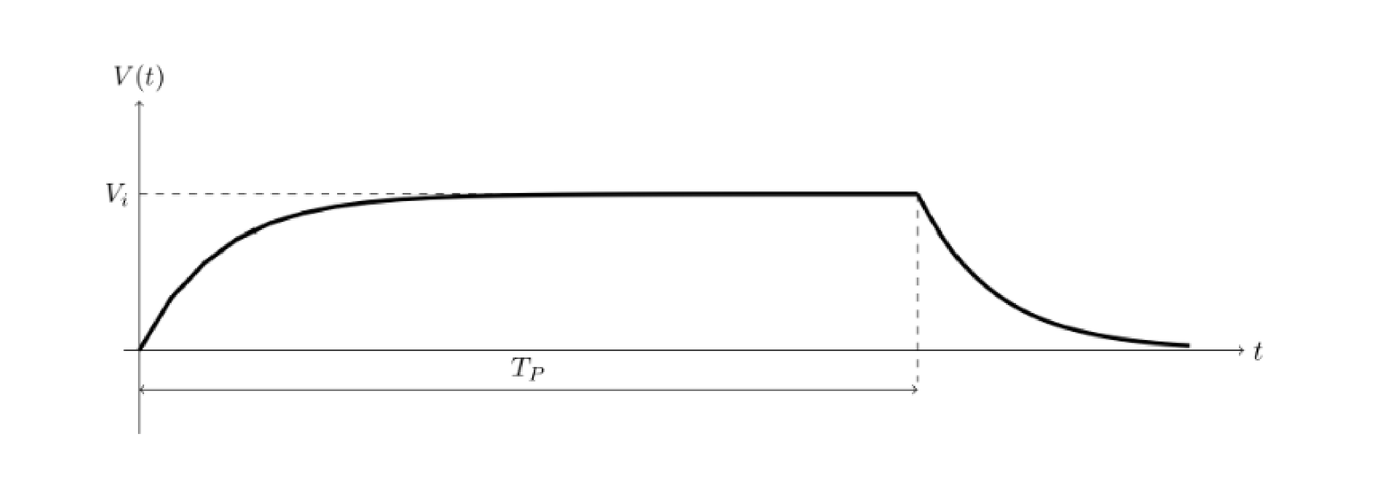
Tutorial – 05



Signal:	Preamplifier:
T_P=10ms	White noise unilateral spectral density $\sqrt{S_{N,U}} = 50\text{nV}/\sqrt{\text{Hz}}$
rise time $T_S \approx 0.2T_P$	1/f noise component with f_c=10kHz
amplitude V_i	Bandwidth: f_{PA}=100MHz

A pulse signal featuring an almost rectangular shape comes from a low impedance source and is fed to a preamplifier with the characteristics reported above.

Considering only white noise (by now):

a) Evaluate the minimum amplitude of the signal that can be measured at the output of the preamplifier without using any additional filtering stage.

b) Select a filter that allows you to observe the signal waveform with improved resolution. Evaluate the minimum amplitude of the signal that can be measured in these conditions.

c) Now consider the exact shape of the signal, which has a rising edge described by the following exponential function $V(t) = V_i * \left(1 - e^{-t/T_{SP}}\right)$, with T_{SP}=1ms, and falling edge with the same shape. Select a filter to improve the precision in the measurement of the signal amplitude and evaluate the minimum amplitude of the signal that can be measured with the selected filter. Before selecting the filter, discuss the characteristics of the optimum filter, then select a filter that can be easily implemented and that is a good approximation of the optimum filter.

Now consider also the 1/f noise component:

d) In the conditions of point b), select an additional filter that allows you to observe the signal waveform limiting the impact of 1/f noise. Evaluate the noise contribution due to the 1/f noise component and therefore the minimum measurable signal amplitude in these conditions.

e) Considering now a known signal waveform as reported in point c), select an additional filter to limit the 1/f noise contribution. Evaluate the noise contribution due to the 1/f noise component and therefore the minimum measurable signal amplitude in these conditions.

A) The minimum measurable signal amplitude is the signal needed to achieve a unitary SNR at the output of the preamplifier, to calculate the SNR, we can use the Equivalent Noise BandWidth (ENBW) of the preamplifier:

$$\boldsymbol{SNR = \frac{V_P}{\sqrt{S_{N,U} \cdot ENBW}} = \frac{V_P}{\sqrt{S_{N,U} \cdot \frac{\pi}{2} f_{PA}}} = 1 \rightarrow V_{P,min} = \sqrt{S_{N,U} \cdot \frac{\pi}{2} f_{PA}} \cong 627\ \mu V}$$

B) We do not know the exact shape, some information might be contained in the signal shape, so we must preserve the shape together with the amplitude, we can use a low-pass RC filter with an appropriate time constant.
The rise of the signal can be expressed as:

$$x(t) = V_P\left(1 - e^{-\frac{t}{T_{SP}}}\right)$$

Considering the rise time T_S as the $10\% \rightarrow 90\%$ rise time, we obtain a time constant of the signal equals to:

$$T_{SP} = \frac{T_S}{\ln(9)} = 0.2 \cdot \frac{T_P}{\ln(9)} \cong 910\ \mu s$$

The bandwidth of the signal is thus equal to:

$$f_{max} = \frac{1}{2\pi T_{SP}} \cong 175\ Hz$$

To preserve the shape of the signal we must place the cut-off frequency at least a decade after f_{max}:

$$\frac{1}{2\pi T_F} = f_{LP} \geq 10 \cdot f_{max} \cong 1.75\ KHz \rightarrow f_{LP} = 2\ KHz$$

For example, we can use $f_{LP} = 2\ KHz$, using the Equivalent Noise BandWidth for a low-pass Rc filter, we obtain an amplitude for the white noise equal to:

$$\sqrt{\sigma_n^2} = \sqrt{S_{N,U} \cdot ENBW_{RC}} = \sqrt{S_{N,U} \cdot \frac{\pi}{2} f_{LP}} \cong 2.8\ \mu V$$

The minimum measurable amplitude is thus:

$$SNR = \frac{V_P}{\sqrt{\sigma_n^2}} = 1 \rightarrow V_{P,min} = \sqrt{S_{N,U} \cdot \frac{\pi}{2} f_{LP}} \cong 2.8\ \mu V$$

C) With the information on the shape, we can improve the results, the signal can be expressed analytically as:

$$x(t) = V_P \cdot b(t) = \begin{cases} V_P \cdot \left(1 - e^{-\frac{t}{T_{SP}}}\right) & for \quad 0 \leq t \leq T_P \\ V_P \cdot e^{-\frac{t-T_P}{T_{SP}}} & for \quad t \geq T_P \end{cases}$$

The optimum filter for the signal described above is the one that matches its shape, the signal acquired is:

$$k_{bb}(0) = \int_{-\infty}^{\infty} b^2(t)dt = \int_0^{T_P} 1 - 2e^{-\frac{t}{T_{SP}}} + e^{-\frac{2t}{T_{SP}}}dt + \int_{T_P}^{\infty} e^{-2\frac{t-T_P}{T_{SP}}}dt \cong T_P - T_{SP}$$

$$SNR = \frac{y}{\sqrt{S_{N,B}}} = \frac{V_P}{\sqrt{\frac{S_{N,U}}{2}}}\sqrt{k_{bb}(0)} = \frac{V_P}{\sqrt{\frac{S_{N,U}}{2}}}\sqrt{T_P - T_{SP}} = 1 \rightarrow V_{P,min} = \sqrt{\frac{S_{N,U}}{2(T_P - T_{SP})}} \cong 373\ nV$$

We can approximate the optimum filter using a Gated Integrator (GI):

$$w_m(t) = rect_{T_G}\left(t - t_m + \frac{T_G}{2}\right)$$

Choosing $T_G = T_P$ to simplify the analysis we obtain:

$$k_{wb}(0) = \int_{t_m}^{t_m+T_P} b(t)dt = \int_{t_m}^{T_P} 1 - e^{-\frac{t}{T_{SP}}}dt + \int_{T_P}^{t_m+T_P} e^{-\frac{t-T_P}{T_{SP}}}dt = T_P + T_{SP} - t_m - 2T_{SP}e^{-\frac{t_m}{T_{SP}}}$$

To find the optimal acquisition instant t_m we must find the maximum of $k_{wb}(0)$:

$$\frac{\partial k_{wb}(0)}{\partial t_m} = 2e^{-\frac{t_m}{T_{SP}}} - 1 = 0 \rightarrow e^{-\frac{t_m}{T_{SP}}} = \frac{1}{2} \rightarrow t_m = T_{SP} \cdot \ln(2) \cong 0.69 \cdot T_{SP} \cong 628\ \mu s$$

Obtaining an acquired signal equal to:

$$k_{wb}(0) = T_P - T_{SP} \cdot \ln(2)$$

Giving us a final SNR of:

$$SNR = \frac{V_P \cdot \left(T_P - T_{SP} \cdot \ln(2)\right)}{\sqrt{\frac{S_{N,U}}{2}T_P}} = 1 \rightarrow V_{P,min} = \sqrt{\frac{S_{N,U} \cdot T_P}{2\left(T_P - T_{SP} \cdot \ln(2)\right)^2}} \cong 380\ nV$$

D) Considering the $\mathbf{1/f}$ noise, and without the information on the exact shape of the signal, we can use a high-pass CR filter to eliminate the baseline, to preserve the shape of the signal, we must impose $f_{HP} \ll \frac{1}{2\pi T_p}$, we can choose for example $f_{HP} = 1\ Hz$, and since $f_{HP} \ll f_{LP}$ we can express the output noise as:

$$\sqrt{\sigma_f^2} = \sqrt{S_{N,U} \cdot f_C \cdot \ln\left(\frac{f_{LP}}{f_{HP}}\right)} \cong 2.7 \cdot \sqrt{S_{N,U} \cdot f_C} \cong 13.8\ \mu V$$

Using the amplitude of the white noise calculated at point **B**, we obtain a total noise amplitude of:

$$\sqrt{\sigma_n^2} = \sqrt{\sigma_f^2 + \sigma_b^2} \cong 14\ \mu V$$

The output signal can be considered unaffected by the high-pass filter, obtaining thus a minimum measurable input value of:

$$SNR = \frac{V_P}{\sqrt{\sigma_n^2}} = 1 \rightarrow V_{P,min} = \sqrt{\sigma_n^2} \cong 14\ \mu V$$

E) Considering the $\mathbf{1/f}$ noise and the information on the shape of the signal, we can use the same high-pass CR filter as before, this time with a gated integrator with a time constant $\tau_G = \tau_P$, and the same displacement calculated at point **C** ($t_m = T_{SP} \cdot \ln(2)$), we can thus express the output noise as:

$$\sqrt{\sigma_n^2} = \sqrt{S_{N,U} \cdot \frac{1}{2T_G} + S_{N,U} \cdot f_C \cdot \ln\left(\frac{\frac{1}{2T_G}}{f_{HP}}\right)} \cong 9.88\ \mu V$$

The output signal is filtered by the gated integrator, giving:

$$y = V_P \cdot \left(T_P - T_{SP} \cdot \ln(2)\right)$$

Obtaining thus a minimum measurable input value of:

$$SNR = \frac{V_P \cdot \left(T_P - T_{SP} \cdot \ln(2)\right)}{\sqrt{\sigma_n^2}} = 1 \rightarrow V_{P,min} = \frac{\sqrt{\sigma_n^2}}{T_P - T_{SP} \cdot \ln(2)} \cong 10.6\ \mu V$$