Tutorial – 11

Exam Text of 25/11/2004 (Problem 3)

The optical transparency of materials at 800nm is to be measured and a laser diode with emitted power P=1mW is used. The laser can be used in continuous wave operation mode (i.e. continuous light is emitted) or with 10% of its optical power sinusoidally modulated at 1MHz (by using a sinusoidal current to drive the diode). A silicon p-i-n photodiode (depleted junction thickness 30um; surface reflectivity coefficient R=0.2) is used. The photodetector is connected to a current preamplifier featuring a wide bandwidth (limited by a single pole at f_{PA}=100MHz) and input-referred current noise with wideband unilateral spectral density Si=(1pA)$_2$/Hz and 1/f noise component with f_C=100Hz. Discuss the guidelines and describe two approaches to be used in the two possible cases. Select a filtering scheme for each case and evaluate quantitatively:

a) The sensitivity that can be obtained, i.e. the minimum optical power that can be measured.
b) The minimum value of the optical coefficient that can be measured.

A) We can calculate the quantum efficiency of the detector as the product of the probabilities of not being reflected at the surface, and of being absorbed in the depletion region, thus, we can write:

$$\eta_D = (1-R)\cdot e^{-\frac{W_N}{L_0}}\cdot\left(1-e^{-\frac{W_D}{L_0}}\right) \cong (1-R)\cdot\left(1-e^{-\frac{W_D}{L_0}}\right) \cong 0.76$$

From the quantum efficiency, we can derive the radiant sensitivity:

$$S_D = \eta_D \cdot \frac{\lambda[\mu m]}{1.24} \cong 0.49$$

Let's consider a continuous wave operation with $P = 1\ mW$, we have both a white and a $1/f$ noise, we can use a chopper/shutter to modulate the signal, with a CDF with two sampling windows, of width T_F and T_G.
To maximize the effect on the $1/f$ we choose $T_G = T_F$, the $1/f$ noise it:

$$\sigma_{1/f} = \sqrt{2.1\cdot S_I \cdot f_C \ln(3)} \cong 15.2\ pA$$

To keep the white noise negligible with respect to the $1/f$ we can write:

$$\sigma_W = \sqrt{2S_I\frac{1}{2T_G}} = \sqrt{\frac{S_I}{T_G}} = \frac{\sqrt{2.1\cdot S_I\cdot f_C\ln(3)}}{10} \rightarrow T_G = \frac{100}{2.1\cdot f_C \ln(3)} \cong 430\ ms \rightarrow \sigma_W = \sqrt{\frac{S_I}{T_G}} \cong 1.5\ pA$$

The minimum measurable current (and thus the total noise) is equal to:

$$I_{P,min} = \sigma_i = \sqrt{\frac{S_I}{T_G} + 2.1\cdot S_I\cdot f_C\ln(3)} \cong 15.3\ pA$$

The minimum measurable optical power is:

$$P_{P,min} = \frac{I_{P,min}}{S_D} \cong 30.5\ pW$$

Let's now consider a modulated wave operation with $P = 0.1\ mW$, in this case the $1/f$ noise can be neglected as we are way beyond the corner frequency, using a LIA with a square-wave reference and an LPF with a cut-off frequency $f_L = 10\ Hz$, we obtain:

$$I_{P,min} = \sigma_i = \sqrt{2}\sqrt{S_I\frac{\pi}{2}f_L} \cong 5.6\ pA$$

The minimum measurable optical power is:

$$P_{P,min} = \frac{I_{P,min}}{S_D} \cong 11.4\ pW$$

B) For the first case (constant wave operation), the minimum measurable transparency is given by the ratio between the minimum optical power measurable by the sensor and the power that arrives on the sample (half the power of the laser due to the presence of the chopper):

$$\boldsymbol{T_{min} = \frac{P_{min}}{P_S} = \frac{P_{min}}{P_{Laser}} \cong 3.05 \cdot 10^{-8}}$$

For the second case (modulated wave operation) we have instead that he minimum measurable transparency is given by the ratio between the minimum optical power measurable by the sensor and the power that arrives on the sample (average power of the sinusoidal wave):

$$\boldsymbol{T_{min} = \frac{P_{min}}{P_S} = \frac{P_{min}}{P_{Laser,sin}} \cong 114 \cdot 10^{-9}}$$

Exam text of 08/02/2018 (Problem 2)

a) Define the radiant sensitivity of a photodetector and how it is possible to write it as a function of the wavelength. Calculate a reasonable value of this parameter at 500nm for a PIN photodiode and for a typical Phototube.

b) Define and explain the meaning of the NEP and Detectivity of a photodiode and a PMT

c) Considering a PMT, calculate the minimum value of the gain G in order to be able to detect single photons on a time window of 5ns.

d) Starting from the random sequence of independent elementary pulses, describe the current shot noise: noise mean, mean square and power and finally power spectrum.

A) The radiant sensitivity of a photodetector is defined as the ratio between the current emitted by the sensor and the optic power, can be expressed as a function of the quantum efficiency:

$$S_D = \frac{I_P}{P_P} = \frac{n_e q_e}{n_P E_P} = \frac{n_e q_e}{n_P E_P} = \frac{n_e}{n_p} \cdot \frac{q_e}{hv} = \eta_D \cdot \frac{q_e \lambda}{hc} = \eta_D \cdot \frac{\lambda[\mu m]}{1.24}$$

Considering a PIN photodiode with $W_n = 100\ nm$, and $W_D = 10\ \mu m$ at $\lambda = 500\ nm,$ and a reflective coefficient $R = 0.2$, we obtain:

$$\eta_D = (1-R)e^{-\frac{W_N}{L_0}}\left(1 - e^{\frac{W_D}{L_0}}\right) \cong 0.72$$

Obtaining a sensitivity of:

$$S_D = \eta_D \cdot \frac{\lambda[\mu m]}{1.24} \cong 0.29\frac{A}{W}$$

Considering instead a classic PMT with an S20 profile ($\eta_D \cong 0.2$) we obtain:

$$S_D = \eta_D \cdot \frac{\lambda[\mu m]}{1.24} \cong 0.08\frac{A}{W}$$

B) The NEP is defined as the minimum optical power measurable by the photodetector, so:

$$NEP = P_{P,min} = \frac{I_{P,min}}{S_D} = \frac{\sigma_i}{S_D} = \frac{\sqrt{2q_e J_b A_D \Delta f}}{S_D}$$

The detectivity is a parameter obtained removing the influence of the area and the bandwidth from the NEP:

$$D^* = \frac{\sqrt{A_D \Delta f}}{NEP} = \frac{S_D}{\sqrt{2q_e J_b}}$$

C) Let's consider a reading circuit made by a resistor and an OPAMP in buffer configuration, let's say the resistance has a value of $\mathbf{50\ \Omega}$ (e.g. a coaxial cable), and the OPAMP has $\sqrt{S_I} = 2\ pA/\sqrt{Hz}$ and $\sqrt{S_V} = 10\ nV/\sqrt{Hz}$, finally, we assume a duration of the experiment equal to $T = 5\ ns.$

Given the PMT's output is a current signal, we can convert the input referred voltage noise into a current noise:

$$\frac{\sqrt{S_V}}{R_L} = 200\frac{pA}{\sqrt{Hz}}$$

The current noise introduced by the resistor is:

$$\sqrt{\frac{4k_BT}{R_L}} \cong 18\frac{pA}{\sqrt{Hz}}$$

Referring the noise to the input of the PMT, we obtain the following SNR:

$$SNR = \frac{i_S}{\sqrt{\left(\frac{S_V}{G^2R_L^2} + 2q_e(i_S + I_D)\right)\cdot\frac{1}{2T}}}$$

Considering a single-photon input, and neglecting the shot noise contribute, we have (assuming a safety margin of $SNR > 3$):

$$i_S = \frac{q_e}{T} \rightarrow\ SNR = GR_L\frac{q_e}{\sqrt{S_V\cdot\frac{T}{2}}} > 3 \rightarrow G > \frac{1}{R_Lq_e}\sqrt{S_V\cdot\frac{T}{2}} = 187\,500 = 1.87\cdot 10^5$$

To check the hypothesis made we must verify that:

$$2q_e(i_S + I_D) \ll \frac{S_V}{G^2R_L^2} \rightarrow i_s + I_D \ll \frac{S_V}{2q_eG^2R_L^2} \cong 3.55\ pA$$

Considering instead a digital approach (single photon counting), we have that we can approximate the SER of the PMT as a triangle, we obtain that:

$$q_e\cdot G = \frac{I_{peak}\cdot T_{SER}}{2}$$

Assuming a width of the Single Electron Response equal to $T_{SER} = T = 5\ ns$, to preserve the shape of the signal we need a bandwidth of:

$$\tau = \frac{T_{SER}}{2} \rightarrow BW = 10\cdot\frac{1}{\tau} \cong 4\ GHz$$

Giving us a noise equal to:

$$\sigma_i = \sqrt{\frac{S_V}{R_L^2}\cdot\frac{\pi}{2}BW} \cong 15.7\ \mu A$$

To keep a reliable system, I_{peak} must be at least equal to $3\sigma_i$:

$$I_{peak} > 3\sigma_i \rightarrow \frac{2q_eG}{T_{SER}} > 3\sigma_i \rightarrow G > \frac{3\sigma_iT_{SER}}{2q_e} = 736\,000 = 7.36\cdot 10^5$$