Tutorial – 12

Exam Text of 25/06/2018 (Problem 2)

SPAD PHOTODIODE	PREAMPLIFIER
Area=0.007 mm^2 Detection efficiency @500nm= 50%; Dark counts=10 cps	- Load Input Resistance R_L=1kΩ - Load Input Capacitance C_L=2 pF - Current Noise (unilateral) at amplifier input $\sqrt{S_{iA}} = 1pA/\sqrt{Hz}$ - Voltage Noise (unilateral) at amplifier input $\sqrt{S_{vA}} = 1nV/\sqrt{Hz}$

We want to study the Earth surface by measuring the time of flight of pulses sent from a satellite in a low orbit (200km from the ground). To perform this measurement, a square laser pulse (pulse width = 1us) at λ=500nm is used, while an APD is used for detection.

1) Discuss and select the most appropriate repetition frequency of the laser. Being able to choose among different APDs, discuss the main features of an APD that could be successfully exploited in this measurement and calculate its detection efficiency and sensitivity.

2) Assuming that the selected APD has a dark current of 1pA and considering the preamplifier with the above reported characteristics, select a suitable filter for this measurement and evaluate the minimum power of a SINGLE laser pulse that can be measured.

3) Due to the movement of the satellite along its orbit, it is possible to make measurements with a maximum duration of 1s. Discuss how the previous measure can be improved and calculate the minimum power that can be measured in this new scenario.

4) How would the situation change if the SPAD with the above reported characteristics was used to replace the APD? Following this approach, calculate the minimum power that can be measured in this case.

A) To correctly detect the individual pulse, we have to have a time distance between two consecutive pulses greater than $T_P = 1\ \mu s$, we can choose for example $2T_P$.
The time of flight is:

$$T_{flight} = 2 \cdot \frac{d}{c} \cong 1.33\ ms$$

We must choose a repetition time greater than $T_P + T_{fligh} \cong T_{flight}$, this means a frequency smaller than:

$$f < \frac{1}{T_{flight}} \cong 750\ Hz$$

To introduce a safety margin, we can use $f = 700\ Hz$.
To properly detect the backscattered pulse, we need an APD with an high detection efficiency at $\lambda = 500\ nm$, considering an APD with $R = 0.2$, $w_N = 100\ nm$ and $w_D = 5\ \mu m$ we obtain:

$$\eta_D = (1-R) \cdot e^{-\frac{w_N}{L_0}} \cdot \left(1 - e^{-\frac{w_D}{L_0}}\right) \cong 0.72$$

Obtaining the following radiant sensitivity:

$$S_D = \eta_D \cdot \frac{\lambda[\mu m]}{1.24} \cong 0.29$$

Let's also consider an APD gain $G = 100$, an excess noise factor $F = 2.5$, and a dark current $I_D \cong 10\ pA$

B) Given the shape of the signal, we can use a Gated integrator (assuming the availability of a sync signal), in this case, we have the following sources of noise:

Noise due the resistor R_L:

$$\sqrt{S_R} = \sqrt{\frac{4k_bT}{R_L}} \cong 4\frac{pA}{\sqrt{Hz}}$$

The input referred current noise of the OPAMP can be expressed as a voltage source:

$$\sqrt{\frac{S_I}{R_L^2}} \cong 1\frac{pA}{\sqrt{Hz}}$$

The shot noise introduced by the dark current:

$$\sqrt{2q_eI_DG^2F} \cong 89\frac{fA}{\sqrt{Hz}}$$

The shot noise introduced by the dark current is negligible with respect to the electronic noise.

Neglecting the effect of the shot noise introduced by the signal, we have a total noise spectrum of:

$$\sqrt{S} = \sqrt{S_I + \frac{S_V}{R_L} + S_R} \cong 4.3\frac{pA}{\sqrt{Hz}}$$

If we use a gated integrator with an integration window $T_G = T_P = 1\ \mu s$ we obtain:

$$\sigma_i = \sqrt{S \cdot \frac{1}{2T_G}} \cong 3\ nA$$

This implies a minimum measurable current of:

$$I_{P,min} = \frac{\sigma_i}{G} \cong 30\ pA$$

The shot noise introduced by the minimum measurable current is:

$$\sqrt{2q_eI_{P,min} \cdot G^2F} \cong 489\frac{fA}{\sqrt{Hz}}$$

And is negligible with respect to the other noise sources.

The minimum measurable optical power is equal to:

$$P_{P,min} = \frac{I_{P,min}}{S_D} \cong 105\ pW$$

C) Using a boxcar integrator, we can improve the SNR of the system, assuming an observation window of $T_W = 1\ s$ and the repetition rate of before $f = 700\ Hz$, we need a BI with a time constant equal to:

$$\tau = \frac{T_W}{5} \cdot T_G \cdot f \cong 140\ \mu s$$

The SNR improvement is equal to:

$$IF = \sqrt{2 \cdot \frac{\tau}{T_G}} \cong 16.7$$

This implies a minimum measurable current of:

$$I_{P,min} = \frac{I_{P,min,0}}{IF} \cong 1.8\ pA$$

The minimum measurable optical power is equal to:

$$P_{P,min} = \frac{I_{P,min}}{S_D} \cong 6\ pW$$

D) Using the proposed SPAD instead than the APD, we can work in single-photon counting mode, the SNR is:

$$SNR = \frac{N_S}{\sqrt{N_S + N_D}}$$

The dark events in the observation windows are:

$$N_D = n_D \cdot T_P \cong 10^{-5}\ events$$

Thus, the minimum number of observable events is:

$$SNR = \frac{N_S}{\sqrt{N_S + N_D}} \cong \sqrt{N_S} = 1 \rightarrow N_S = 1$$

The minimum measurable optical power is equal to:

$$P_{P,min} = \frac{1}{\eta_D} \cdot \frac{E_P}{T_G} \cong 800\ fW$$

Otherwise, we could have calculated the minimum measurable current as a single electron in the window:

$$I_{P,min} = \frac{q_e}{T_W} \cong 160\ fA$$

And the sensitivity of the detector:

$$S_D = \eta_D \cdot \frac{0.5}{1.24} \cong 0.2$$

Giving us a minimum power of:

$$P_{P,min} = \frac{I_{P,min}}{S_D} \cong 800\ fW$$