FORMULARY of RF SYSTEM

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Contents

1 Introduction to Rf signals

1.1 Main features

Fundamental condition:	$\frac{B}{f_0} \ll 1$
Standard Representation:	$s_{RF}(t) = V_M \cos(\omega_0 t + \phi(t))$
In phase / In quadrature:	$s_{RF}(t) = V_I(t)\cos(\omega_0 t) \pm V_Q(t)\sin(\omega_0 t)$
Exponentional form:	$s_{RF}(t) = Re\{V_M(t)e^{j\phi(t)}e^{j\omega_0 t}\}$
Phasor Notation:	$V_c = V_M(t)e^{j\phi(t)} = V_I(t) + jV_Q(t)$

1.2 Power Transfer

With a specified line of transmission

Power of the source (S):	$P_{in} = \frac{1}{2}Re(v_S {i_S}^*)$
Power to the load (L):	$P_{out} = -\frac{1}{2}Re(v_L {i_L}^*)$
Available power:	$P_{av} = \frac{1}{8}\frac{\lvert V\rvert^2}{Re(Z_S)}$
Matching condition:	$Z_L = {Z_S}^* \Rightarrow P_L = P_{av}$

Gain

General definition:	$G_P = \frac{P_{out}}{P_{in}}$
Transducer Power Gain:	$G_T = \frac{P_{out}}{P_{av,in}} = 4\frac{Re(Z_s)}{Re(Z_L)}\left\lvert\frac{V_{out}}{V_{in}}\right\rvert^2$
Available Power Gain:	$G_A = \frac{P_{av,out}}{P_{av,in}} = \frac{ReZ_S}{Re(Z_{out})}\left\lvert\frac{V_{out}}{V_{in}}\right\rvert^2$
Relationship between gain definitions:	$G_T \ll G_A \quad (or \;\; G_P)$
Cascade of 2-port:	$G_P = G_{P1}G_{P2}G_{P3}\dots G_{Pn}$ $G_A = G_{A1}G_{A2}G_{A3}\dots G_{An}$
If port interfaces are matched:	$G_T = G_{T1}G_{T2}G_{T3}\dots G_{Tn}$

1.3 Noise Generation

Bandwidth:	ΔF
Noise Power:	$P_N = KT\Delta F$

With a 2-ports device?

Noiseless port:	$P_{av,out} = G_{av}P_{av,in} = G_{av}P_N = G_{av}KT\Delta F$
Noise Power of the device:	$P_D = KT_{eq}\Delta F G_{av}$
Noisy port:	$P_{av,out} = G_{av}P_N + P_D = G_{av}KT\Delta F + P_D$

Attenuator

Attenutation:	$A = \frac{1}{G_T}$
Equal temperature:	$T_{eq} = (A-1)T_0$

Noise Figure (of a 2-port)

Definition 1:	$NF = \frac{\text{Output Noise Power}}{\text{Output NP for noiseless ideal model}}$
Definition 2:	$NF = 1 + \frac{T_{eq}}{T_0}$
Definition 3:	$NF = \frac{(S/N)_{in}}{(S/N)_{out}}$
Cascade of 2-ports:	$NF_{tot} = NF_1 + \frac{NF_2 - 1}{G_{av1}} + \frac{NF_3 - 1}{G_{av1}G_{av2}}$

Mixer

- The frequency of the input is different from the output frequency. Exploiting a local oscillator (usually sinusoidal) the mixer can change the frequency of the carrier

Down conversion	$f_{Rf,out} \ll f_{Rf}$
Up conversion	$f_{Rf,out} \gg f_{Rf}$

- There are two channels: RF and Image Frequency. Both theese channels count to the overall noise of the device. This model is called DSB. The alternative is known as SSB, because only one contribution atually matters.
- Remember that Mixers are passive devices, hence they introduce attenutation, not gain!

DSB:	$T_{DSB} = T_0^{'}(\frac{A_c}{2} - 1)$
SSB:	$T_{SSB} = 2T_0^{'}(\frac{A_c}{2} - 1)$
NF:	$NF = \frac{(S/N)_{in}}{(S/N)_{out}} = 2 + \frac{T_{SSB}}{T_O^{'}}$

2 Antennas and Link equation

2.1 Directional properties

Radiation intensity:	$U(\theta, \phi)$
R.I. for isotropic antennas:	$U = \frac{P_{rad}}{4\pi}$
Power density:	$S_R = \frac{dP_{rad}}{dS} = \frac{1}{2} Re\{\bar{E} x \bar{H}^*\} = \frac{1}{R^2} U(\theta, \phi)$
Directivity Gain:	$D(\theta, \varphi) = \frac{U(\theta, \varphi)}{P_{rad}/4\pi} = \frac{Radiation}{Isotropic\ \ Radiation}$
Maximum of D:	$D_{max} = \frac{U(\theta_{max}, \varphi_{max})}{P_{rad}/4\pi}$
Directivity function:	$f(\theta, \varphi) = \frac{D(\theta, \varphi)}{D_{max}}$
Direction of maximum propagation:	$f(\theta, \varphi) = 1$

2.2 Transmitting Antenna

Avilable power for the antenna:	P_T
Efficiency factor:	$\eta = \frac{Re\{Z_R\}}{Re\{Z_r\} + R_P}$
Radiated power/Electrical power:	$P_{rad} = \eta P_T$
Power Density:	$S_R = \frac{P_{rad}}{4\pi R^2} D_{max} f(\theta, \varphi)$
Gain:	$G = \eta D_{max}$
Power Density 2:	$S_R = \frac{P_T}{4\pi R^2} \eta D_{max} f(\theta, \varphi) = \frac{P_T}{4\pi R^2} G f(\theta, \varphi)$
ERP - effective radiated power:	$ERP = P_T G$
Beamwidth (for a dish antenna):	$\Delta\theta = 2\theta = 2\cos^{-1}\left(1 - \frac{2}{D_{max}}\right)$
Fields intensity:	$S_R = \frac{1}{2} \frac{\sqrt{\epsilon_r}}{Z_w} \lvert E \rvert^2 = \frac{1}{2} \frac{Z_w}{\sqrt{\epsilon_r}} \lvert H \rvert^2$

Evaluation of Gain by means of the directivity function
{Suppose that Σ is a hemisphere}

Radiated Power:	$P_{rad} = \iint_{\Sigma} S_R(R, \theta, \varphi)\, d\Sigma = \frac{P_{rad} D_{max}}{4\pi R^2} \iint_{\Sigma} f(\theta, \varphi)\, d\Sigma$
Infinitesimal element of the surface:	$d\Sigma = R^2 \sin(\theta) d\theta d\varphi$
Hence we obtain:	$D_{max} = \frac{4\pi}{\iint_{\Sigma} f(\theta, \varphi)\, d\Sigma \frac{1}{R^2}} = \frac{G}{\eta}$

2.3 Receiving Antenna

Received Power: $P_R = S_R A_e g(\theta, \varphi)$

Effective Area/Gain: $\frac{G}{A_e} = \frac{4\pi}{\lambda^2}$

Effective Area (dish antenna, fixed area): $A_e = e_a \frac{1}{4}\pi d^2$

2.4 Noise at the antenna output/Link Budget

Friis equation: $P_R = S_R A_e g(\theta, \varphi))$

Friis equation 2: $P_R = P_T \cdot G_t \cdot f(\theta, \varphi) \cdot G_r \left(\frac{\lambda}{4\pi R}\right)^2 \cdot g(\theta, \varphi)$

System SNR: $SNR_{sys} = \frac{P_r}{K \; T_{sys} \; B}$

Remember that B is the signal band and T_{sys} is the equivalent temperature of the system. Now, the Friis equation under condition of optimal direction of propagation leads to the following expression:

System SNR: $SNR_{sys} = \frac{P_t G_t}{KB}\left(\frac{\lambda}{4\pi R}\right)^2 \frac{G_R}{T_{sys}} = \frac{P_{ERP}}{L_f}\frac{1}{KB}\left(\frac{G_R}{T_{sys}}\right)$

Data Rate Limits

Shannon's theorem - max data rate: $R_{max} = C = B\log_2(1 + SNR)$

Bit Error Rate: $BER = \frac{E_b}{N_0} = \frac{energy \;\; per \;\; unit}{noise \;\; spectral \;\; density}$

Time to receiving one bit: $T_b = \frac{1}{R}$

Received Power: $P_R = \frac{E_b}{T_b} = E_b R$

Since System SNR... $SNR_{sys} = \Big(\frac{E_b}{N_0}\Big)\Big(\frac{R}{B}\Big) = P_{ERP}\frac{1}{L_f}\frac{1}{KB}\Big(\frac{G_R}{T_{sys}}\Big)$

Hence... Rate: $R = \frac{P_{ERP}}{E_b/N_0}\frac{1}{KL_f}\Big(\frac{G_R}{T_{sys}}\Big)$

Roll-off coefficient: α

M-QAM modulation: $B = \frac{R}{\log_2 M}(1 + \alpha)$

System SNR: $SNR_{sys} = \frac{E_b}{N_0}\Big(\frac{log_2 M}{1 + \alpha}\Big)$

Satellite Link - System SNR: $SNR_{sys} = \frac{1}{\frac{1}{SNR_u} + \frac{1}{SNR_d}}$

3 Characterization of non-linearity in RF systems

2-port memoryless network I/O (ideal model: no active devices):	$v_{out} = a_0 + a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3 \dots$
"Many tones" signal (example):	$V = A\cos(\omega_0 t) + A\cos(\omega_2 t) + A\cos(\omega_3 t) + \dots$
Numbero of tones:	N
Power of each tone:	$P_T \propto \frac{A^2}{2}$
Average Power (signal):	$P_{AV}\ (or\ P_m) = NP_T = N\ \ KA^2$
Peak Power (signal):	$P_P = K(NA)^2 = N^2 P_T$
Peak Factor (of the signal):	$F = \frac{P_P}{P_{AV}}$
Peak Envelope Power:	$PEP = \frac{1}{2}\|max(V_{ev})\|^2$

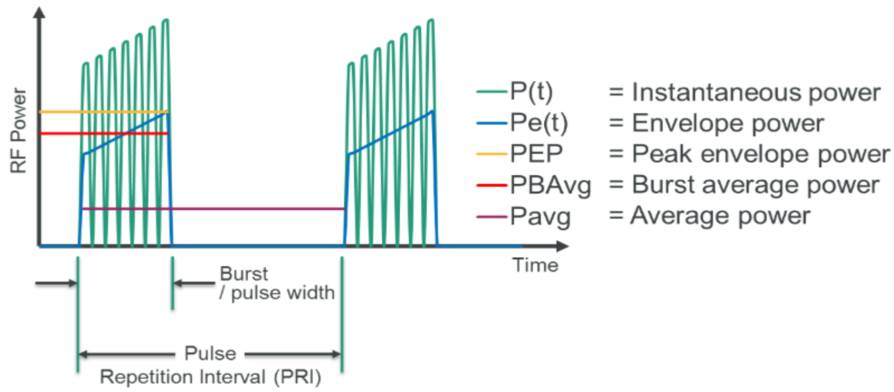


Figure 1: Peak and Average Measurements of a Pulse Modulated Signal

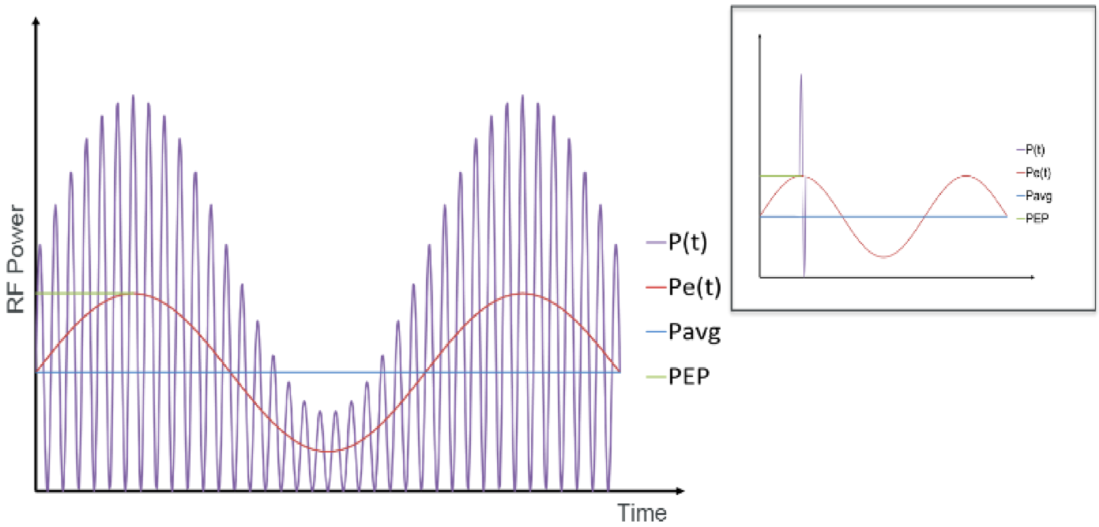


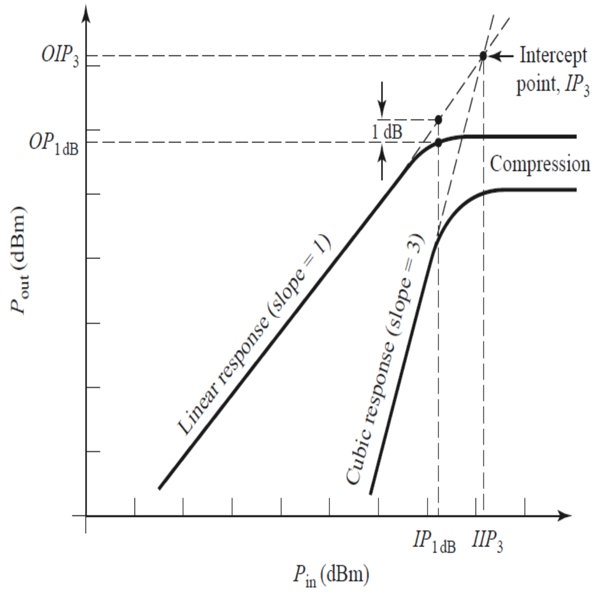
Figure 2: Another Example of Instantaneous vs. Peak vs. Average Powers

The "1dB compression Power" gauges the power of the signal after the effect of distortion. This compression is caused by the non-linearity of the device. More precisely, if we were to write down the explicit expansion of the generic I/O relationship considered by the professor, we would observe that the coefficient of the first term changes with respect to the linear regime.
The easiest way to represent a RF signal is a 2-tones signal. However with more tones at the input occurs the problem of intermodulation: all the combinations of the main frequencies appears at the output. Every combination has a order:

General combination: $n\omega_1 \pm m\omega_2$

Order of the combination: $n + m$

We want to filter the signal in order to cancel out the undesidered contributions produced by the distorsion. However, *if we approximate the system at the third order*, we would observe that two combinations are too close to the main frequencies to be neglected by the filtering. These two frequencies are $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$. Now, plotting the output power of the main frequencies and of the third order components, *under linear approximation*, we observe that the two lines met at some point. This point is called *Third Order Intercept Point*.
To summarize:



{Consider the following equations expressed in decibel}

Relation between power values: $P_{2\omega_2-\omega_1} = 3P_{\omega_1} - 2IP_3$

Average power of the main fequencies: $P_m = P_{\omega_1} + 3$

Average power of the intermodulation 3rd order components: $P_{int} = P_{2\omega_2-\omega_1} + 3$

Carrier-to-intermodulation ratio: $CI_{dBm} \simeq 2IP_3 - 2P_m + 6$

Specifying the output power of a PA for a 2-tones signal

$$PEP = N^2 P_{T,monotone} = 4P_{T,monotone} \sim 2P_{m,2tones}$$

Hence (in dB):

$$PEP = P_{m,2tones} + 3$$

and

$$CI = 2IP_3 - 2(PEP - 3) + 6$$

Eventually:

$$IP_3 = P_{1dB} + \Delta p$$

Where $\Delta_p = 10$.

Backoff:	$BO = \frac{CI}{2} - \Delta_p - 3$
Peak factor (different peak power) 2:	$F = \frac{PEP}{P_m} = N$
Power conversion efficiency:	$PAE = 100 \times \frac{P_{RFsignal,load} - P_{RFsignal,input}}{DCpower}$
Adjacent Channel Power Ratio:	$inf = \frac{P_{out}}{P_{inf}}$
Degradation of digital signal powers:	$EVM(\%) = 100 \times \sqrt{\frac{P_{error}}{P_{reference}}}$

Mixer: parameters referred to the input - P_{LO} is typically above P_{1db}

$$P_{i,1dB} = P_{1dB} - G_{dB}$$

$$IIP_3 = P_3 - G_{dB}$$

G_{dB} is the linear gain expressed in decibel

4 Receiver Architecture

4.1 Theory introduction

Suppose to have the most generic scheme possible. We recognise three parts: the *front-end*, the *intermediate frequency* and the *passband.*

In the RF front-end we find the receiving antenna (usually a dish type one), the first microwave filter (to cancel the image band), the power amplifier (that's usually a LNA, "low noise amplifier"), the second microwave filter (to cancel the noise of the amplifier, absent in presence of LNA) and the Mixer (which is connected to a local oscillator, used to tune the signal at the right frequency). We must point out that ...

The local oscillator spectrum should be a straight line, but the phase fluctuactions leads to a broadening of the spectrum that affects the SNR_{sys}.

The LNA has a double benefit: it amplifies the signal and decrease the noise (see *Equivalent model*)

Actually, the tuning operation that makes possible the frequency conversion presents a difficult choice. Imaging the spectrum, we can easily understand that while an high IF provides high performances even with a simple image filter (thanks to the high level of rejection), low IF are easier to achieve, work at low frequency and most of all keep the cost low (on the other hand the channel bandwidth is limited).
For this reason the Double Conversion Receivers were designed: in the scheme there are two mixers, one high IF and one low IF. The first one filters out the image band (fixed IF), meanwhile the other one complete the conversion at the right intermediate frequency (variable IF). Nowadays we have also Image Reject Mixers, devices that can handle the problem of the image band without involving an expensive filter instead.

[Another possible design] Direct Conversion receiver: there are not IF filters. The complexity is minimized, it's easier to achieve but it's more susceptible to noise and distorsion.

4.2 Some parameters

Analog Receiver

Sensitivity:	S
SNR minimum:	$SNR = \dfrac{S}{KT_{eq}B}$

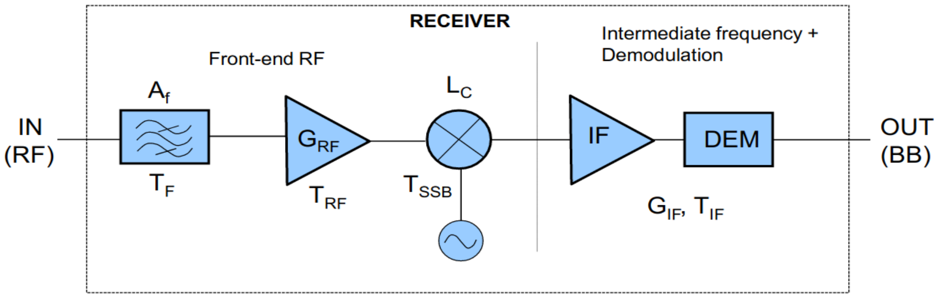
Digital Receiver

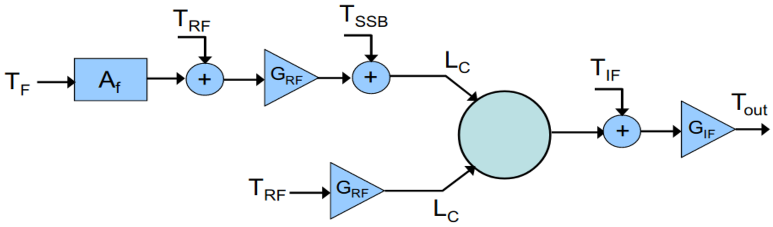
Sensitivity:	$S \Longleftrightarrow E_b$
SNR:	Do propagate $\dfrac{E_b}{N_0}$ back to the input
Dynamic Range, minimum:	S
DR, maximum (a possible definition):	$DR_{dB} = \dfrac{2}{3}(IIP_{3,dBm} + 3 - S_{dBm})$
Overall IP at the input of LNA:	$\left(\dfrac{1}{IIP_3}\right)^2 = \left(\dfrac{1}{IIP_{3,LNA}}\right)^2 + \left(\dfrac{G_{LNA}}{IIP_{3,mixer}}\right)^2$
Spurious responses:	$f_{RF} = \dfrac{nf_{LO} \pm f_{IF}}{m} \quad m, n = 1, 2, 3 \ldots$

When the sensitivity, DR (or P_{max}) and the ratio R over B (bit rate over passband) are assigned:

Receiver Budget Analysis: $$SNR = \frac{S}{KT_{rec}B} = \frac{E_b}{N_0}\frac{R}{B}$$

4.3 Evaluation of the receiver noise





- 1st: the total noise at the output:

$$T_{out} = \Big\{\Big[\Big(\frac{T_F}{A_f} + T_{RF}\Big)G_{RF} + T_{SSB}\Big]\frac{1}{L_C} + \frac{T_{RF}G_{RF}}{L_C} + T_IF\Big\}G_IF$$

- 2nd: the noise at the output is propagated back to the input:

$$T_{REC} = T_{out}\frac{L_C A_f}{G_{RF}G_{IF}}$$

- 3rd: every contribution is made explicit

$$T_{REC} = T_F + 2A_f T_{RF} + \frac{Af(T_{SSB} + L_c T_{IF})}{G_{RF}}$$

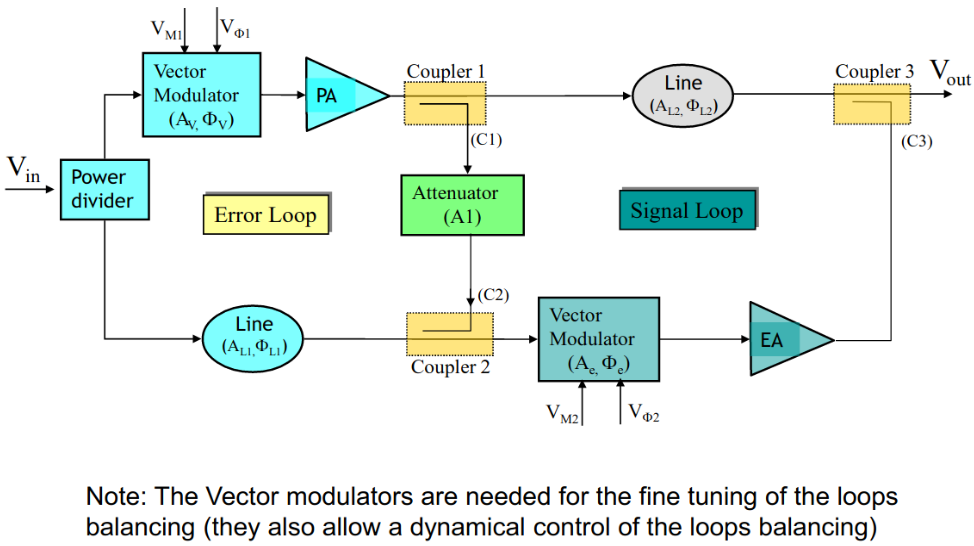
5 Transmitter and Feedforwrding

5.1 Theory introduction

In the generic scheme of a transmitter we start with a local oscillator, followed by a modulator. The first one provides the carrier signal. It must be frequency stable, since the quality of the signal at the receiver depends also on the degradation introduced by its phase noise. Next, the modulator introduces the information. In the case of digital signal the modulator combine the I- and Q-components ("In phase" and "Quadrature") to modulate both the amplitude and the phase of the RF signal.
Then there are a filter, a power amplifier and the transmitting antenna at the end.

For what concerns the non-lineariy of the the transmitter, the amplified signals presents obviously some kind of distortion and there will be also spurious components affecting adjacent channels. The parameters used to gauge the livel of distortion are, for istance, the BER, the ACPR and the EVM. Note that signal with a constant envelope can tolerate more distortion than signal with both phase and amplitude variation. However, since the constant envelope signal don't have a high spectral efficiency, they are replaced with more efficient modulation schemes, which presents an envelope variation. This substitution leads to the importance of designing *linearizers*, especially when we want to amplify more channels.

5.2 Feedforward



- Mathematical model of the vector modulators:

$$V_{out} = K_m V_0 \Big[\Big(1 + \frac{V_I}{V_0}\Big) + j\Big(1 + \frac{V_Q}{V_0}\Big)\Big] V_{in} = A e^{j\phi} V_{in}$$

- Mathematical model of the coupler:

$V_{coup} = \gamma V_{in}$ $\quad V_{out} = -j\beta V_in$ $\quad \gamma \ll \beta$

lossless condition: $\gamma^2 + \beta^2 = 1$ (first approxiamtion)

I/O relationship: $V_{out} = \gamma V_e - j\beta V_{in}$ and $V_e \propto -V_{coup}$

- Delay lines: $\tau_g = \dfrac{1}{2\pi}\dfrac{\partial \phi}{\partial f}$

Analysis

1st hp:	lossless condition (ideally $\beta \approx 1$)
2nd hp:	amplifying with 0 delays associated
3rd hp:	we refer to amplitude

We must derive the loop equations from the scheme of the circuit.
Balance condition of the error loop:

$$-A_s + G_M C_1 - A_1 - C_2 = -A_e \Rightarrow G_M = (A_s - A_e) + C_1 + A_1 + C_2$$

Balance condition of the signal loop:

$$-C_1 - A_1 - C_2 + G_E - C_3 = 0 \Rightarrow G_E = C_1 + A_1 + C_2 + C_3$$

No ideal error amplifier (EA):
Suppose to test the linearizer with a two-tones signal. Now, the error amplifier introduces distortion. Therefore, at the output, two more components appear in the spectrum.

Distortion cancellation condition:	$P_E - C_3 = P_{M,D}$
PA Carrier-to-intermodulation:	$CI_M = P_M - P_{M,D}$
EA Carrier-to-intermodulation:	$CI_E = P_E - P_{E,D}$

$$\Rightarrow CI_E + P_{E,D} - C_3 = P_M - CI_M$$

$$\Rightarrow CI_M + CI_E = P_M - (P_{E,D} - C_3)$$

Finally, looking at the output spectrum, we can derive the expression to compute the "total" Carrier-to-intermodulation of the FeedForward:

$$CI_{ff,\inf} = P_M - (P_{E,D} - C_3) = CI_M + CI_E \qquad [dB]$$

Unbalanced loop:
A missmatch error of the loop amplitude δA and/or phase $\delta\phi$ determines a reduction of the distortion suppression.

Amplitude:	$B = 10^{-\dfrac{\delta A}{20}}$
Phase:	$e^{j\delta\phi}$

Reduction of distortion suppression:

$$S = -20\log\left(\left|\frac{V_\varepsilon}{V_{rif}}\right|\right) = -10\log(1 + B^2 - 2B\cos(\delta\phi))$$

Note: the use of S is trivial, take a look to the exerscises. However, to better understand its conceptual meaning, we can consider S1 the reduction of distortion suppression measured at the output of the first loop, that is, the output of C2. It represents a fraction of the reference signal that is added to the distorsion entering the second loop.
In other words: ideally at the output of the error loop we would have two components related to the distortion introduced by the first power amplifier (in the upper path). But, in the case of an unbalanced loop, the merging made by the second coupler don't cancel totally the signal power and some residual enters in the

second loop (called unsuppresed for obvious reasons). S_1 is the value of difference between the signal power and these possible residuals (not sure).

Feedfoorward efficiency

Main Amplifier:	efficiency η_M, Ouput Power P_M, CI ratio CI_M
Error Amlifier:	efficiency η_M, Output Power P_E
Output Couplee:	coupling C_3, Through-path coupling L_3

In natural units:

$$f_M = 10^{\frac{-CI_M}{10}}, \ \ l_3 = 10^{\frac{-L_3}{10}}, \ \ c_3 = 10^{\frac{-C_3}{10}}$$

Now, the efficiency is:

$$\eta_{ff} = \frac{\eta_M \eta_E P_M (1 - c_3)}{\eta_M P_M + \eta_M f_M l_3 \frac{P_M}{c_3}} = \frac{\eta_M \eta_E P_M (1 - c_3)}{\eta_E c_3 + \eta_M f_M 81 - c_3}$$

6 Useful observations

From the exercises

- Do keep in mind that the noise introduced by the devices doesn't depend on the power signal. Thus, once that T_{device} has been computed, it doesn't change in the following. On the other hand SNR depends on the power signal, so it changes with respect to the point that we are considering.
- We can calculate the SNR in every point of a system with the overall equivalent temperature referred to the input. The parameter T_{eq} sums up all the contributions in one single value associated to the input.
- The attenuation L_f, in decibel, is computed as $10log\left(\frac{4\pi R}{\lambda}\right)^2$, where R is the distance between the antennas.
- Since the mixer converts both the signal frequency and the image one, there are two channels which means two power values. Both of them are amplified by the LNA. Note that this situation could be avoided if there was a filter cancelling the image band (?).
- If the mean power of intermodulation at some point is imposed equal to system noise power:

  $$P_{int} = KT_{eq}B \Rightarrow SNR = \frac{P_{m,r}}{P_{int}}$$

  Hence:

  $$SNR_{dB} = P_{m,received,dBm} - P_{int,dBm} = CI$$
- (Computation of D_{max}) If it is not differently specified, the range of θ is $[0; \pi]$, while φ varies between 0 and 2π
- (Computation of T_{eq}) The image band noise equivalent temperature must be considered when the front end involves at east an amplifier and a mixer in that order. Normally the image band is amplified by the LNA and converted by the MIXER, but if there is a filter in between we can assume that the image band is erased by the filtering.
- The *half power beamwidht*, known with the symbol θ_{3dB}, is defined as two times the diffrence between θ_{max} and the direction $\bar{\theta}$ along which the power is a half (-3dB).

7 Trasmission Lines

7.1 Basic Concepts

Wave function:	$v^+(z) = (V_0 e^{j\omega_0 t})e^{-\gamma z}$
Phasor of the voltage:	$V_0 e^{j\omega_0 t}$
Propagation constant:	$\gamma = \alpha + j\beta$
Attenuation coefficient:	α
Phase constant:	$\beta = \frac{2\pi}{\lambda_0} = \frac{\omega}{\nu}$
Characteristic impedance:	$Z_c = \sqrt{\frac{L}{C}}$
Useful relationship:	$\alpha = \frac{1}{2}\frac{R}{Z_c} + \frac{1}{2}GZ_c, \quad \beta = \omega\sqrt{LC}, \quad vel = \frac{1}{\sqrt{LC}}$

Therefore:

$$Z_c = \frac{1}{vC} = vL$$

7.2 Voltage, currents, reflection coefficient

Voltage along the line:	$V(z) = v^+(z) + v^-(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$
Current along the line:	$I(z) = i^+(z) + i^-(z) = I_0^+ e^{-j\beta z} + I_0^- e^{+j\beta z}$
Characteristic impedance	$Z_c = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-}$

Reflection Coefficient:

$$\Gamma(z) = \frac{\text{Refleced wave}}{\text{Incident wave}} = \frac{Z_L - Z_c}{Z_L + Z_c} = \frac{V_0^- e^{+j\beta z}}{V_0^+ e^{-j\beta z}} = \frac{V_0^-}{V_0^+} e^{+j2\beta z} = \Gamma_0 e^{+j2\beta z} = \Gamma_0 \exp\left(j2\frac{2\pi}{\lambda} z\right)$$

- the magnitude is constant and always less than 1 if there is a passive load;
- the phase is periodic, period $= \lambda/2$.

Now, the previous expressions are written using the reflection coefficient:

Voltage:	$V(z) = V^+(z)[1 + \Gamma(z)]$
Voltage Magnitude:	$\lvert V(z)\rvert = \lvert V_0^+\rvert \lvert(1 + \Gamma e^{+j2\beta z})\rvert$
Maximum:	$\rightarrow 1 + \lvert\Gamma_0\rvert$
Minimum:	$\rightarrow 1 - \lvert\Gamma_0\rvert$
Voltage Standing Wave Ratio:	$VSWR = \frac{V_{max}}{V_{min}}$
VSWR = 0:	*Perfectly matched*
VSWR = ∞:	*Totally missmatched*

Impedance (normalized): $\frac{Z(z)}{Z_0} = \frac{1+\Gamma(z)}{1-\Gamma(z)}$

Inverting the latter relation: $\Gamma(z) = \frac{(Z(z)/Z_c)-1}{(Z(z)/Z_c)+1} = \frac{Z(z)-Z_c}{Z(z)+Z_c}$

Impedance (function of the load): $Z_{in} = \frac{V_{in}}{I_{in}} = Z_c \frac{Z_L + jZ_c \tan(\beta L)}{Z_c + jZ_L \tan(\beta L)}$

Note that L is the distance from the load. Indeed, probably, the reference system is the most important thing to point out, start with defining which point corresponds to z=0.

7.3 Stubs to modelize circuit components

These particular trasmission lines are used to obtain inductive and capacitative devices. We aproximate inductors with short circuit stubs and capacitors with open circuit stubs.
It's imposed that $d < \lambda/4$

Inductor replacing stub: $Z_c = \frac{X_{inductor}}{\tan(\beta_0 d)} = \frac{\omega_0 L}{\tan\left(\frac{\omega_0}{\nu} d\right)}$

Capacitor replacing stub: $Y_c = \frac{B_{capacitor}}{\tan(\beta_0 d)} = \frac{\omega_0 C}{\tan\left(\frac{\omega_0}{\nu} d\right)}$

7.4 Summary of some T.L parameteres and Smith Chart

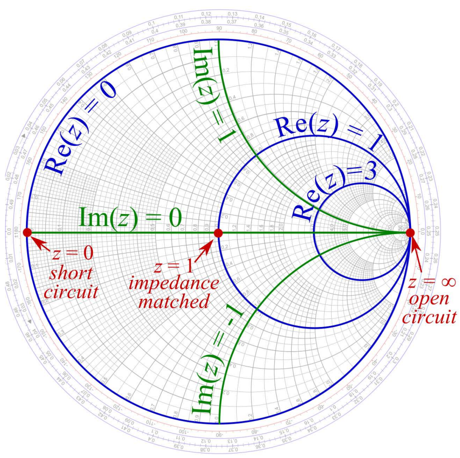
SUMMARY

Cut-off: f_c

Phase velocity: $v_f = \frac{c}{\sqrt{\epsilon_{r,eff}}} \frac{1}{\sqrt{1-\left(\frac{f_c}{f_0}\right)}}$

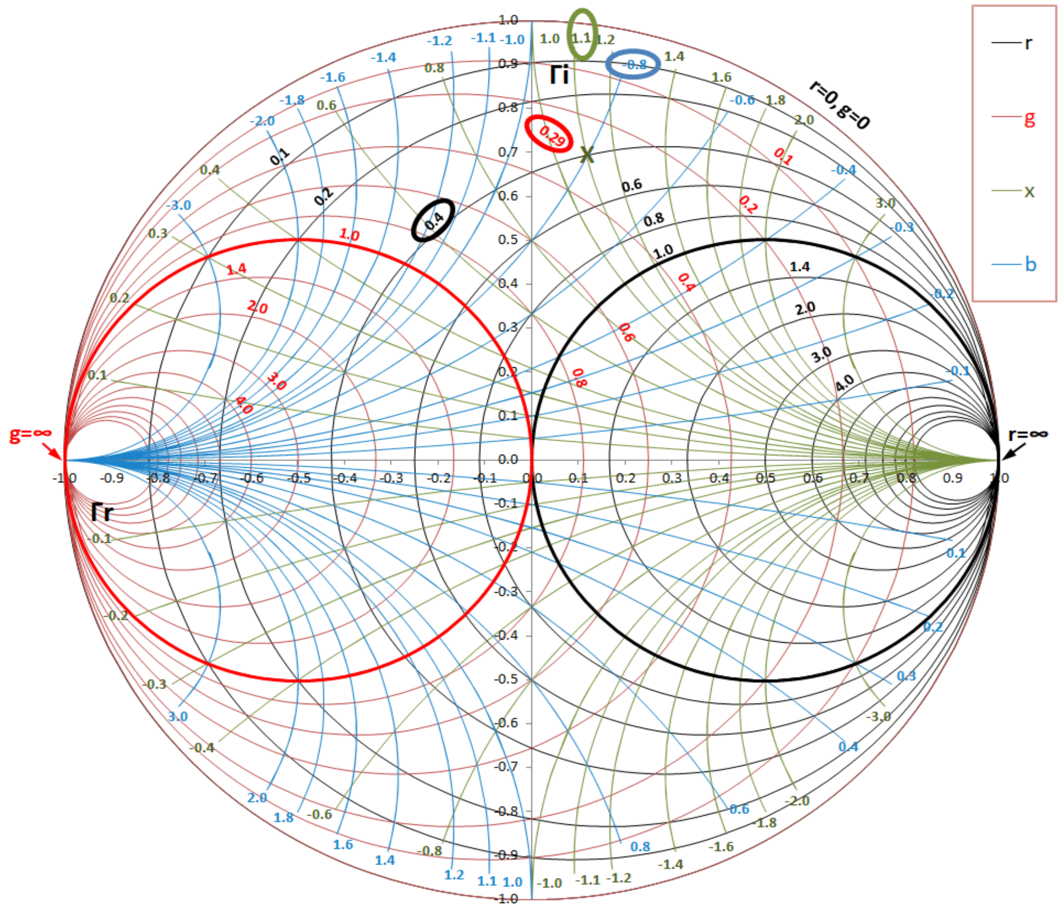
Wavelength: $\lambda = \frac{V_f}{f} = \frac{\lambda_0}{\sqrt{1-\left(\frac{f_c}{f_0}\right)}}$

SMITH CHART:



We use the Smith Chart to analyze the paramaters characterizing the transmission lines. In practice we will use a specific executable file for Windows, offered by the professor, which consists into an electronical Smith Chart that exploits Matlab runtime.

- The Smith Chart is used to represent the complex coefficient of reflection. If the line of trasmission is closed on a passive load, the magnitude of the reflection coefficient is less than 1, thus the vector can be rappresented, in polar coordinate, inside a circle of unit radius. Since does exist a biunivocal relationship between reflection coefficient and the normalized impedance/admittance seen at the interface, the coordinate of the reflecion coefficient corresponds to the real and imaginary part of the impedance (admittance).
- The normalization consists into dividing the real values of the loads, stubs, capacitors or inductors by the reference characteristic value of the transmission line, e.g. $Z_{norm} = {}^{Z_{load}}/_{Z_{line}}$;
- The picture above shows the real and imaginary parts of specific complex values of z. However it's possible to draw also some admittances values, but their "circles" are reflected, i.e. the position of short circuit and open circuit are reversed:



Remind that: $Y = g + jb$ and $Z = r + jx$.

USE OF THE SMITH CHART

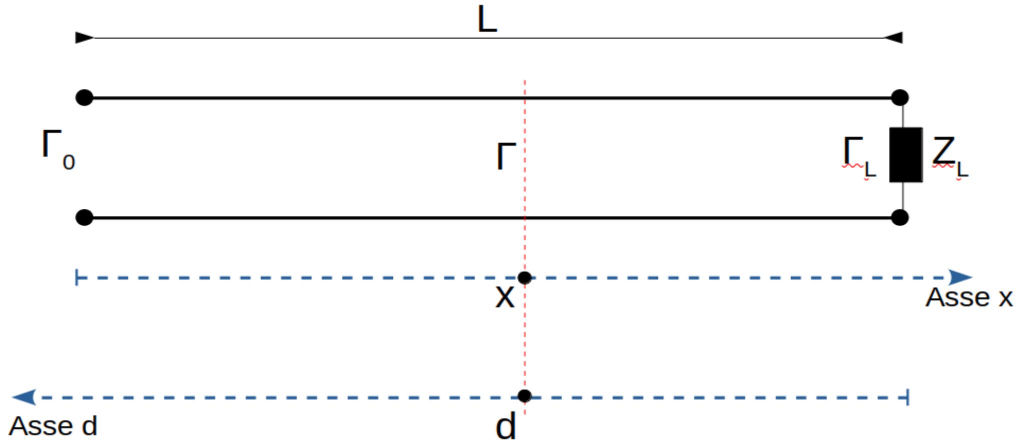
- The reflection coefficient is a vector that has origin in the center of the Smith Chart. The magnitude of this vector is:

$$|\Gamma| = \sqrt{Re\{\Gamma\}^2 + Im\{\Gamma\}^2}$$

- However, in general, we write this coefficient as a function of the spatial coordinate z that represent the distance from a reference interface:

$$\Gamma(z) = \Gamma_{ref} e^{\pm j2z\beta}$$

- The picture below shows a simple line of trasmission and two different computation to obtain the reflection coefficient in the middle:



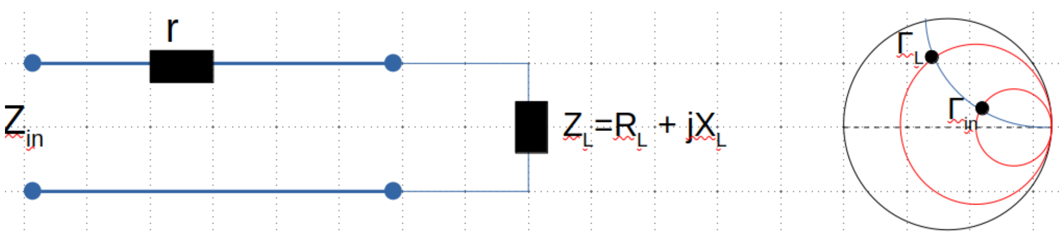
$$\Gamma = \Gamma_0 \ \ e^{+j2\beta x} = \Gamma_L \ \ e^{-j2\beta d}$$

- Particular values for Γ:

Values	Meaning
$\Gamma = 0$	Matching condition satisfied at the port
$\Gamma = 1$	Open circuit if working with impedances, Short circuit with admittances
$\Gamma = -1$	Short circuit for impedances, Open Circuit with admittances

- To understand which points correspond to the optima of the voltages, since V(z) depends on $\Gamma(z)$, we must find the maximum and minimum real values of Γ exploiting the Smith Chart and the right expression for the reference considered.

- In presence of a series (or a parallel) of two devices we can compute the total impedance (admittance) summing the impedances (admittances) of the devices. What happen on the Smith Chart?



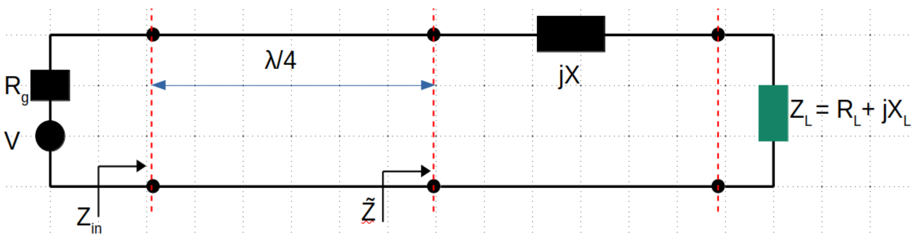
It's easy to conclude that $Z_{in} = (r + R_L) + jX_L$. This is known as displacement at costant reactance, because the imaginary part of the load doesn't change as shown in the right picture. In the case of an imaginary device, like a capacitor or a inductor, we would observe a displacement at costant resistance. In this latter case, the refelction coefficient would move on the same red circle.

- When we move on the trasmission line, we're rotating on the Smith Chart: referring to the reflection coefficient, moving from the source to the load means rotating counterclockwise; viceversa moving from the load to the source means rotating clockwise.
- Every distance on the trasmission line corresponds to an arc of circonference on the Smith Chart. The relation is the following: $d \rightarrow 2\beta d$.

7.5 Matching Networks

Available Power at the source (Recall):	$P_{av} = \frac{1}{8}\frac{V^2}{Re(Z_s)}$
Conjugate Matching Condition:	$Z_{in} = Z_s^*$ and $Z_{out} = Z_L^*$
1st hypothesis:	*The Matching Network is lossless*
2nd hypothesis:	*If conjugate matching is satisfied at one port, it's also satisfied at any other section*

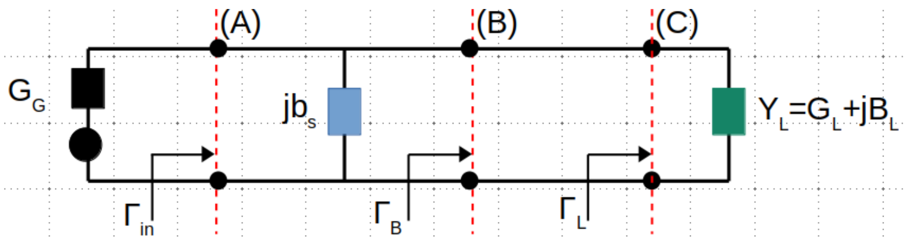
$\lambda/4$ line - Impedance inverter



Equivalent load impedance:	$\tilde{Z} = R_L + j(X_L + X)$
General relation:	$Z_{in} = \frac{Z_0^2}{\tilde{Z}}$
Matching condition:	$Z_{in} = Z_s^* = R_g$

Note that this setup works only if Z_{in} is a real value, *because we've supposed real the intrinsic resistance of the generator*. Under this condition the only solution is provided if $X = -X_L$. This result remains the same with admittances, because the reasoning doesn't change.

Single Stub Matching



1. To satisfy the matching condition we must obtain $\Gamma_{in} = 0$. Since we're going to use the admittances, another way to express the matching at the input port is $Y_s = Y_{in}^*$. However $Y_s = G_g$, that is a real value $\rightarrow Y_{in}$ must be real as well.

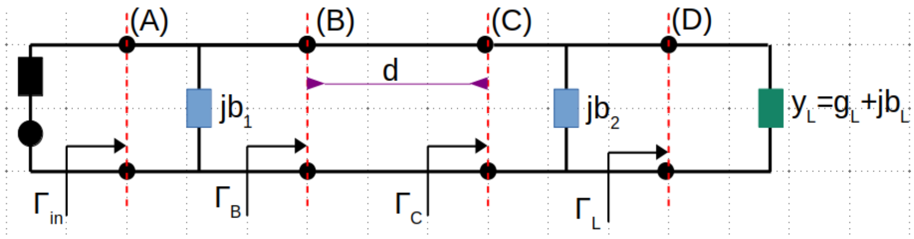
2. If we suppose that the characteristic admittance of the line is $Y_{ch} = G_g$, whhen we normalize the paramaters we obtain what follows:

$$g_G = 1 \qquad y_{in} = 1 \text{ (match. cond.)} \qquad \Gamma_{in} = 0$$

3. The stretch within interfaces C and B corresponds to a rotation at $|\Gamma_L| =$ constant. We move clockwise untill we find the interception point with the circle defined by $Re(y) = 1$ (it's a necessary condition because $g_G = 1$, real value). Therefore, measuring the arc of this displacement, we find also the actual value of the distance $d = x(C) - x(B)$. We can write, in general, $\Gamma_B = 1 + jb_B$.

4. If we were using admittances, at this point we should sum the susceptance b_s (B_s normalized) to the admittance at interface B. Anyway, since we're working with the reflection coefficient on the Smith Chart, to obtain the matching condition requested, we must reach the center of the chart. In other words, we move at costant conductance: $g = 1$ defines the circle on which we are trasforming the Γ_B into $\Gamma_{in} = 0$. It's evident that b_s must be b_b.

Double-Stub Matching



1. It's asked to achieve the conjugate matching at the input port, the interface A. So we want $\Gamma_{in} = 0$ and we suppose $g_G = 1$.

2. Let's start observing the solving relation: $y_{in} = 1$. In general $y_{in} = g_B + j(b_B + b_1)$, thus we need suitable values to satisfy the equation $g_B + j(b_B + b_1) = 1$:

$$\begin{cases} g_B = 1 \\ b_B = -b_1 \end{cases} \qquad \Rightarrow \Gamma_B \text{ must be on the circle } g = 1$$

3. Observing the line, it's evident that Γ_B is obtained through a displacement at constant magnitude on the Smith Chart: the distance d "transforms" Γ_C into Γ_B. However, Γ_C is obtained with a shift on the circle $g = g_L$, starting from Γ_L.

4. Γ_C is the key to solve this problem. As a matter of fact, it belongs to the circles $g = g_L$ and, after rotation, $g = 1$. Hence, to find Γ_C we simply rotate the circle $g = 1$ counterclokwise (toward load) by $2\beta d$, obtaining two points of interception which are the possible solutions of this problem, for the paramater Γ_C.

Notes about the choice of the stubs
In order to design the best line, the shorter is the stubs the better is the design. In other words, we need to check if it is shorter a short circuit stub or a open circuit stub. To solve this comparison, with respect to the parameter we're using, it's sufficient to look at the Smith Chart. Moreover, it could be possible that the dual stub is shorter. In this case we may have to change the design from series to parallel, or viceversa.
Remembder that stus have imaginary impedances/admittances.

8 Matrix characterization of electrical netwroks and Microwave Circuits

8.1 Matric characaterization

Mathematical model for a n-port linear circuit:

$$\vec{V} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \dots \\ v_n \end{bmatrix} \qquad \vec{I} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \dots \\ i_n \end{bmatrix} \qquad \Rightarrow \begin{cases} \bar{\bar{Z}} = \dfrac{\vec{V}}{\vec{I}} & \text{impedance matrix} \\ \bar{\bar{Y}} = \dfrac{\vec{I}}{\vec{V}} & \text{admittance matrix} \end{cases}$$

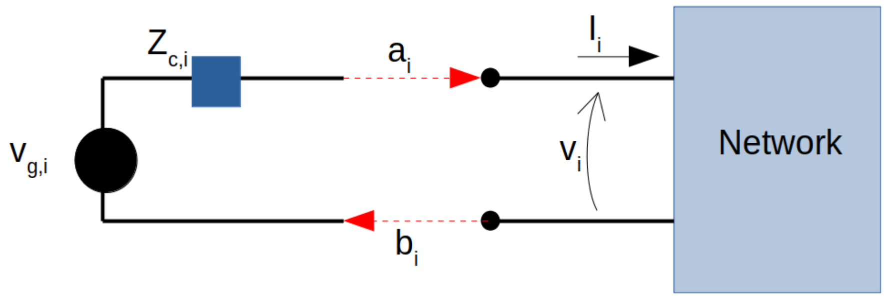
Since $v = i \cdot z$ and assuming currents indipendent paramters, i.e. impressed values, we can write:

$$\begin{cases} v_1 = z_{1,1}i_1 + z_{1,2}i_2 + \cdots + z_{1,N}i_N \\ v_2 = z_{2,1}i_1 + z_{2,2}i_2 + \cdots + z_{2,N}i_N \\ \dots \\ v_3 = z_{N,1}i_1 + z_{N,2}i_2 + \cdots + z_{N,N}i_N \end{cases}$$

Properties of $\bar{\bar{Z}}$ and $\bar{\bar{Y}}$

- The response, or excitation, remains the same when the ports are exchanged. In other words, for a reciprocal N-ports netwrok: $z_{i,j} = z_{j,i}$, and $y_{i,j} = y_{j,i}$. Hence $\bar{\bar{Z}}$ and $\bar{\bar{Y}}$ are symmetric.
- Passivity: assuming absence of sources inside the network, the sum of the power flowing through all the ports must be positive:
$$P = \frac{1}{2}Re(V_1I_1^* + V_2I_2^* + \cdots + V_NI_N^*) \geq 0$$
- Lossless: in case of no dissipation the averall power must be equal to 0. This means, for example in terms of impedances, that $z_{i,j} = -(z_{j,i})^*$. Moreover, i we suppose that the network is reciprocal, we conclude that all the elements must be imaginary.

Microwave linear circuits



Incident Power Wave:	$\frac{1}{2}\lvert a_i\rvert^2$
Reflected Power Wave:	$\frac{1}{2}\lvert b_i\rvert^2$
Available Power:	$P_{AV} = \frac{1}{2}\lvert a_i\rvert^2 = \frac{\lvert V_{g,i}\rvert^2}{8Re\{Z_{c,i}\}} = \frac{\lvert Z_{c,i}I_i + V_i\rvert^2}{8Re\{Z_{c,i}\}}$
Coefficient a_i	$a_i = \frac{V_i + Z_{c,i}I_i}{2\sqrt{Re\{Z_{c,i}\}}} = I_i\frac{Z_i + Z_{c,i}}{2\sqrt{Re\{Z_{c,i}\}}}$
Coefficient b_i	$b_i = \frac{V_i - Z_{c,i}^*I_i}{2\sqrt{Re\{Z_{c,i}\}}} = I_i\frac{Z_i - Z_{c,i}^*}{2\sqrt{Re\{Z_{c,i}\}}}$

Due to the linearity of the circuit, we can express as follows the relation beatween incident and reflected waves:

$$\begin{cases} b_1 = s_{11}a_1 + s_{12}a_2 + \dots + s_{1,N}a_N \\ b_2 = s_{21}a_1 + s_{22}a_2 + \dots + s_{2,N}a_N \\ \dots \\ b_N = s_{N1}a_1 + s_{N2}a_2 + \dots + s_{N,N}a_N \end{cases}$$

Exploiting matrix form we write $\bar{b} = \bar{\bar{S}} \cdot \bar{a}$. $\bar{\bar{S}}$ is known as **scattering matrix**:

$$\bar{\bar{S}} = \begin{pmatrix} s_{11} & \cdots & s_{1N} \\ \vdots & \ddots & \vdots \\ s_{N1} & \cdots & s_{NN} \end{pmatrix}$$

- $s_{ii} = \frac{b_i}{a_i}\Big|_{a_{k \neq i}=0}$: reflection coefficient at port i when the other ports are connected to their reference impedances $Z_{c,j}$, i.e. matching condition satisfied
- $s_{ij} = \frac{b_i}{a_j}\Big|_{a_{k \neq i}=0}$: reflection coefficient between port j and i, with all other ports matched.

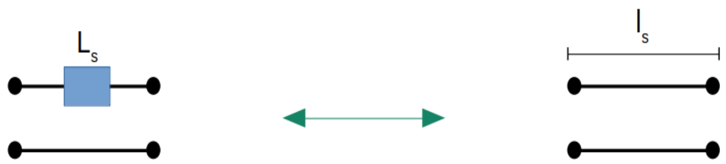
Note that $|s_{i,j}|^2$ is the transducer power gain between the two ports.

For a reciprocal network $\bar{\bar{S}}$ is symmetric. For a lossless netwrok $\bar{\bar{S}}$ is unitary

In general, real microwave circuits are interconnections of components whose size is comparabale with the wavelength at the operation frequency.
To properly represent the junction between components, i.e. to take into account the power dissipation, we must use the scattering matrix. Indeed, using an ideal model as reference is not possible: the physical discontinuities excite high order modes; that means losses, even if they don't propagate because they are "below" the cut-off.
Usually the parameters of the scattering matrix are evaluated with a software, that simulates the propagation of electromagnetic waves inside specified structures.

Examples of scattering matrix
In the previous chapter it is explained how to change the input impedance using matching networks. We can realize them using lumped components or equivalent stubs, and in chapter 7.3 are shown the proper relationships to design this replacement. Now, these relations hold also when we use scattering materix, recalling that the following lines of trasmissions are 2-ports network, with two inputs and two outputs. The scattering matrices require 4 parameters:

Series Inductor

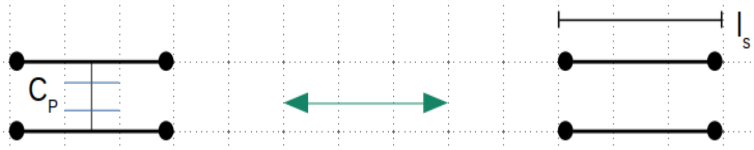


$$Y_L = \begin{pmatrix} \frac{1}{j\omega L_s} & -\frac{1}{j\omega L_s} \\ -\frac{1}{j\omega L_s} & \frac{1}{j\omega L_s} \end{pmatrix} \qquad Y_s = \begin{pmatrix} \frac{1}{jZ_c \tan(\beta l_s)} & -\frac{1}{jZ_c \tan(\beta l_s)} \\ -\frac{1}{jZ_c \tan(\beta l_s)} & \frac{1}{jZ_c \tan(\beta l_s)} \end{pmatrix}$$

$$Y_L \approx Y_s \text{ and } \beta l_s \approx 0 \Rightarrow \tan(\beta l_s) \approx \sin(\beta l_s) \approx \beta l_s$$

$$\Rightarrow \omega L_s = Z_c \beta l_s \Rightarrow L_s = \frac{Z_c l_s}{\nu}$$

Shunt Capacitor



$$Z_c = \begin{pmatrix} \frac{1}{j\omega C_p} & -\frac{1}{j\omega C_p} \\ -\frac{1}{j\omega C_p} & \frac{1}{j\omega C_p} \end{pmatrix} \qquad Y_s = \begin{pmatrix} \frac{1}{jY_c \tan(\beta l_s)} & -\frac{1}{jY_c \tan(\beta l_s)} \\ -\frac{1}{jY_c \tan(\beta l_s)} & \frac{1}{jY_c \tan(\beta l_s)} \end{pmatrix}$$

$$Z_L = Z_s \text{ and } \beta l_s \approx 0 \Rightarrow \tan(\beta l_s) \approx \sin(\beta l_s) \approx \beta l_s$$

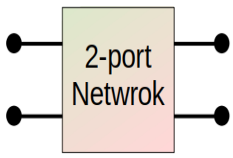
$$\Rightarrow \omega C_p \approx Y_c \beta l_s \Rightarrow C_p = \frac{Y_c l_s}{\nu}$$

8.2 Eigenvectors and Eigenvalues of a matrix

Memo: an eigenvector, of a matrix $\bar{\bar{S}}$, is a particular vector $\bar{V}$ that doesn't change direction if multiplied by $\bar{\bar{S}}$. The result of this product can be obtained also as $\lambda \cdot \bar{V}$, where λ is the eigenvalue. Hence, by definition, the eigenvalues are the solution of the equation $\det[\bar{\bar{S}} - S_\lambda \bar{\bar{U}}] = 0$.

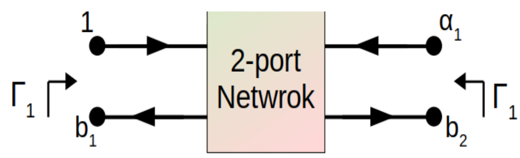
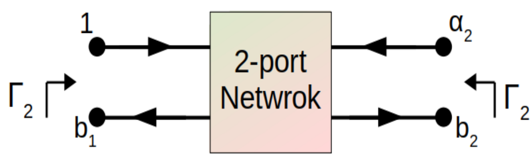
Peoperties: if a N-port is excited with a vector of currents representing a eigenvector of Z, you see the same impedance at all ports, and its value is just the eigenvalue. We obtain the eigenvalues looking for symmetry axis in the network that allow us to identify suitably defined circuits, teh eigencircuits.

Example: 2-ports:



$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$

Suppose to have 2 eigenvectors and that the largest n element for each x_i is 1:

Eigenvector:	2-port network	Eigenvalue:
$x_1 = \begin{bmatrix} +1 \\ \alpha_1 \end{bmatrix}$		$\frac{b_1}{1} = \frac{b_2}{\alpha_2} = \Gamma_1$
$x_2 = \begin{bmatrix} 1 \\ \alpha_2 \end{bmatrix}$		$\frac{b_1}{1} = \frac{b_2}{\alpha_2} = \Gamma_2$

- Eigenvector 1:

$$\begin{cases} b_1 = s_{11} \cdot 1 + s_{12} \cdot \alpha_1 \\ b_2 = s_{12} \cdot 1 + s_{22} \cdot \alpha_2 \\ \dfrac{b_1}{1} = \dfrac{b_2}{\alpha_2} = \Gamma_1 \end{cases}$$

- Eigenvector 2:

$$\begin{cases} b_1 = s_{11} \cdot 1 + s_{12} \cdot \alpha_2 \\ b_2 = s_{12} \cdot 1 + s_{22} \cdot \alpha_2 \\ \dfrac{b_1}{1} = \dfrac{b_2}{\alpha_2} = \Gamma_2 \end{cases}$$

- Matrix elements:

$$s_{11} = \frac{\alpha_1\Gamma_2 - \alpha_2\Gamma_1}{\alpha_1 - \alpha_2} \qquad s_{12} = s_{21} = \frac{\Gamma_1 - \Gamma_2}{\alpha_1 - \alpha_2} \qquad s_{22} = \frac{\alpha_1\Gamma_1 - \alpha_2\Gamma_2}{\alpha_1 - \alpha_2}$$

- Relationship between eigenvalues of $\bar{\bar{S}}$, $\bar{\bar{Y}}$ and $\bar{\bar{Z}}$:

$$S_\lambda = \frac{Z_\lambda - Z_0}{Z_\lambda - Z_0} = \frac{Y_0 - Y_\lambda}{Y_0 + Y_\lambda} \qquad Z_\lambda = Z_0\frac{1 + S_\lambda}{1 - S_\lambda} = \frac{1}{Y_\lambda}$$

The evaluation of the eigenvector is generally possible only exploiting some software for simulation. However if the network is symmetric ($s_{11} = s_{22}$)we can actually deduct its eigenvalues. With respect to the previous examples, to obtain the same reflection coefficient at port 1 and port 2, we need to impose the following condition: $\alpha_1 = \alpha_2 = \pm 1$.

$$\text{Eigenvector 1 (Even): } \begin{bmatrix} +1 \\ +1 \end{bmatrix} \qquad s_{11} = s_{22} = \frac{\Gamma_e + \Gamma_o}{2} \qquad \Gamma_e = s_{11} + s_{12}$$

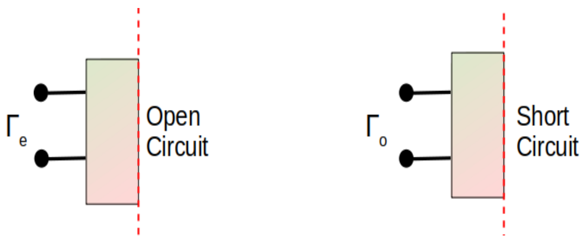
$$\text{Eigenvector 2 (Odd): } \begin{bmatrix} +1 \\ -1 \end{bmatrix} \qquad s_{12} = s_{21} = \frac{\Gamma_e - \Gamma_o}{2} \qquad \Gamma_o = s_{11} - s_{12}$$

Observing the vertical symmetry axis we can prove that with an even excitation (two equal inputs) we can replace it with an open circuits. Viceversa, an odd excitation corresponds to a short circuit.
Even excitation: $b_1 = s_{11} \cdot 1 + s_{12} \cdot 1$
Odd excitation : $b_1 = s_{11} \cdot 1 + s_{12} \cdot (-1)$
We know that the total power flowing in, supposing a lossless network, must return back. Hence, by definition:

$$\Gamma_e = 1 \qquad \Gamma_o = -1$$

Hence, from the equations introduced above we derive that

$$b_1 = 1 = 1 \cdot 1 + 0 \cdot 1$$



9 Directional Couplers and Coupled TEM Lines

9.1 Directional Couplers

A directional coupler is a 4-port Netwrok, but:

1. all ports are matched on the reference load: $s_{11} = s_{22} = s_{33} = s_{44} = 0$
2. two pairs of ports are uncoupled: typically (1,3) and (2,4) $\rightarrow s_{i,j} = 0$

The coupling, C, is a parameter defined as the lowest scattering paramter. In our case, let's suppose that is s_{13}.

$$C = |s_{13}|^2 \rightarrow C_{dB} = -20\log(|s_{13}|)$$

For example, considering a reciprocal and lossless network, assuming (1,4) and (2,3) uncoupled:

- The unitary condition is imposed on the first port:

$$|s_{11}|^2 + |s_{12}|^2 + |s_{13}|^3 + |s_{14}|^2 = 1 \Rightarrow |s_{12}|^2 + C = 1 \rightarrow |s_{12}| = \sqrt{1-C}$$

- Second port: same reasoning...

$$|s_{21}|^2 + |s_{22}|^2 + |s_{23}|^3 + |s_{24}|^2 = 1 \Rightarrow |s_{21}|^2 + |s_{24}|^2 = |s_{12}|^2 + |s_{24}|^2 = 1 \rightarrow |s_{24}|^2 = |s_{13}|^2 = C$$

- Third port:

$$|s_{31}|^2 + |s_{32}|^2 + |s_{33}|^3 + |s_{34}|^2 = 1 \Rightarrow |s_{31}|^2 + |s_{34}|^2 = C + |s_{24}|^2 = 1 \rightarrow |s_{24}| = s_{12} = \sqrt{1-C}$$

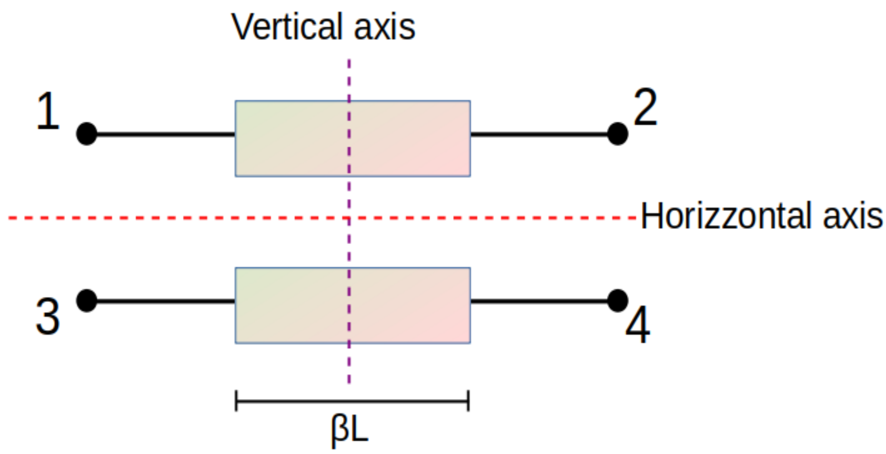
Always keeping in mind that we've found the value of magnitude, supposing that every scattering parameter is positive, a clear overview of the corresponding matrix is the following:

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & \sqrt{1-C} & \sqrt{C} & 0 \\ \sqrt{1-C} & 0 & 0 & \sqrt{C} \\ \sqrt{C} & 0 & 0 & \sqrt{1-C} \\ 0 & \sqrt{C} & \sqrt{1-C} & 0 \end{bmatrix}$$

A further implication of lossless condition, when network is reciprocal, is that the outputs are in quadrature.

9.2 Coupled TEM Lines

Consider the case of two trasmission lines placed close to each other.



The electromagnetic wave propagates along the overall line in two modes: one called even and the other called odd.Each of them is characterized by its own impedance (Z_e, Z_o).
Our goal is computing the four port matrix ($\bar{\bar{Z}}$, or $\bar{\bar{Y}}$, or $\bar{\bar{S}}$).
To do that we suppose that we have equal lines, with symmetric structure.

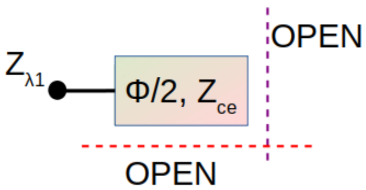
- We can modelize each mode with an ideal wall:

- With reference to $\bar{\bar{Z}}$, the exciting currents for each eigenvector result:

$$I_{\lambda 1} = \{+1,+1,+1,+1\} \quad \Rightarrow \text{H. Axis: Mag, V. Axis: Mag}$$
$$I_{\lambda 1} = \{+1,-1,+1,-1\} \quad \Rightarrow \text{H. Axis: Mag, V. Axis: Ele}$$
$$I_{\lambda 1} = \{+1,+1,-1,-1\} \quad \Rightarrow \text{H. Axis: Ele, V. Axis: Mag}$$
$$I_{\lambda 1} = \{+1,-1,-1,+1\} \quad \Rightarrow \text{H. Axis: Ele, V. Axis: Ele}$$

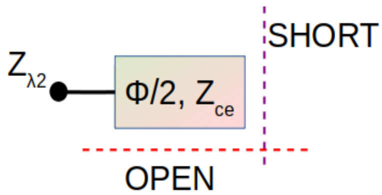
If two ports share the same sign we can replace the symmetry axis with a magnetic wall. Viceversa, if the values are opposite we raplace the symmetry axis with electric wall.

Eigenvalue $Z_{\lambda 1}$:



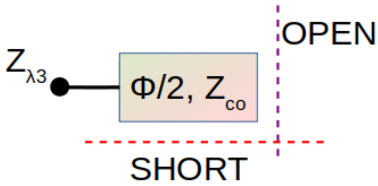
$Z_{\lambda 1} = -jZ_{ce} \cot(\phi/2)$

Eigenvalue $Z_{\lambda 2}$:



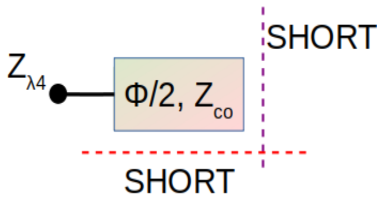
$Z_{\lambda 2} = jZ_{ce} \tan(\phi/2)$

Eigenvalue $Z_{\lambda 3}$:



$Z_{\lambda 3} = -jZ_{co} \cot(\phi/2)$

Eigenvalue $Z_{\lambda 4}$:



$Z_{\lambda 4} = jZ_{co} \tan(\phi/2)$

- Recalling the definition of the matrix elements:

$$Z_{\lambda 1} = \frac{V_1}{I_{\lambda 1}} = Z_{11} + Z_{12} + Z_{13} + Z_{14}$$

$$Z_{\lambda 2} = \frac{V_2}{I_{\lambda 2}} = Z_{11} - Z_{12} + Z_{13} - Z_{14}$$

$$Z_{\lambda 3} = \frac{V_3}{I_{\lambda 3}} = Z_{11} + Z_{12} - Z_{13} - Z_{14}$$

$$Z_{\lambda 4} = \frac{V_4}{I_{\lambda 4}} = Z_{11} - Z_{12} - Z_{13} + Z_{14}$$

  Hence obtain $Z_{11} = {}^1\!/_4\{Z_{\lambda 1} + Z_{\lambda 2} + Z_{\lambda 3} + Z_{\lambda 4}\}$. In general we can obtain each parameter of the first row as the prodcut of the corresponding current vector by the vector of the eigenvalues: $Z_{1i} = {}^1\!/_4\{\bar{I_{\lambda i}} \times \bar{Z_{\lambda i}}\}$. Compact expressions:

$$Z_{11} = -j\frac{(Z_{ce} + Z_{co})}{2}\cot\phi \qquad Y_{11} = -j\frac{(Y_{ce} + Y_{co})}{2}\tan\phi$$

$$Z_{12} = -j\frac{(Z_{ce} + Z_{co})}{2}\frac{1}{\sin\phi} \qquad Y_{12} = +j\frac{(Y_{ce} + Y_{co})}{2}\frac{1}{\sin\phi}$$

$$Z_{13} = -j\frac{(Z_{ce} - Z_{co})}{2}\cot\phi \qquad Y_{13} = -j\frac{(Y_{ce} - Y_{co})}{2}\cot\phi$$

$$Z_{14} = -j\frac{(Z_{ce} - Z_{co})}{2}\frac{1}{\sin\phi} \qquad Y_{14} = -j\frac{(Y_{ce} - Y_{co})}{2}\frac{1}{\sin\phi}$$

9.3 Special cases

$\underline{\phi = \beta L = 180}$

The eigenvalues of Z are $[0, \infty, 0, \infty]$. Hence those of matrix $\bar{\bar{S}}$ are: $S_{11} = 0, S_{12} = -1, S_{13} = 0 = S_{14}$. Note that the port 2 is completeley uncoupled form the line 1!

Perfect Matching at all ports

There is a value Z_0 for which the ports are all matched ($S_{11} = S_{22} = S_{33} = S_{44} = 0$), there are not reflected waves at the ports.
This vaue does not depend on the lenght: $Z_0 = \sqrt{Z_{ce}Z_{co}}$. How dis we get this value?

The eigenvalules of $\bar{\bar{Z}}$: $Z_\lambda = j\{-Z_{ce}\cot(\phi/2), Z_{ce}\tan(\phi/2), -Z_{co}\cot(\phi/2), Z_{co}\tan(\phi/2)\} = jX_{\lambda i}$. This latter equivalence does NOT mean that each eigenvalue is equal to a corresponding reactance. In this case the letter X is used as variable to write faster the rest of the reasoning.
The eigenvlues of $\bar{\bar{S}}$ are derived:

$$S_{\lambda i} = \frac{jX_{\lambda i} - Z_0}{jX_{\lambda i} + Z_0}$$

For the matching condition: $s_{11} = {}^1\!/_4\{S_{\lambda 1} + S_{\lambda 2} + S_{\lambda 3} + S_{\lambda 4}\} = 0$. Hence tere are only two posssible solutions:

1. :

$$\begin{cases} (S_{\lambda 1} + S_{\lambda 2}) = 0 \\ (S_{\lambda 3} + S_{\lambda 4}) = 0 \end{cases} \Rightarrow \begin{cases} X_{\lambda 1} \cdot X_{\lambda 2} = -Z_0^2 \\ X_{\lambda 3} \cdot X_{\lambda 4} = -Z_0^2 \end{cases} \Rightarrow \begin{cases} Z_{ce}^2 = Z_0^2 \\ Z_{co}^2 = Z_0^2 \end{cases}$$

   Solution not admissible, because Z_{ce} must be different from Z_{co}.

2.

$$\begin{cases}(S_{\lambda 1}+S_{\lambda 4})=0\\(S_{\lambda 2}+S_{\lambda 3})=0\end{cases} \Rightarrow \begin{cases}X_{\lambda 1}\cdot X_{\lambda 4}=-Z_0^2\\X_{\lambda 2}\cdot X_{\lambda 3}=-Z_0^2\end{cases} \Rightarrow \begin{cases}\sqrt{Z_{ce}Z_{co}}=Z_0\\\sqrt{Z_{ce}Z_{co}}=Z_0\end{cases}$$

That is admissible and it's independent on $\phi=\beta L$.

Coupled TEM lines as directional couplers

If all ports are matched, $Z_0=\sqrt{Z_{ce}\cdot Z_{co}}$, assuming the port 4 uncoupled, the maximum value of $(|s_{13}|^2)_{max}$ defines C:

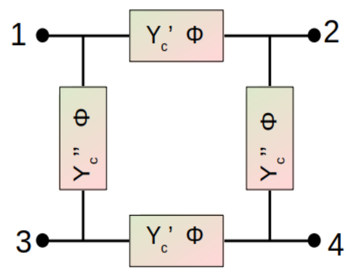
$$C=(|s_{13}|^2)_{max}=|{}^1\!/\!_4(S_{\lambda 1}+S_{\lambda 2}-S_{\lambda 3}-S_{\lambda 4})|^2_{max}=\left|\frac{Z_{ce}-Z_{co}}{Z_{ce}+Z_{co}}\right|^2$$

Moreover for real devices, C varies with frequency:

$$C(\phi)=\frac{C_{max}}{1+(1-C_{max})\cot^2\phi}$$

9.4 Couplers with lumped couplings

1. Branch line



- $\phi=\dfrac{\pi}{2}$, that is $L=\dfrac{\lambda}{4}$
- We define $B_s=Y_c'+Y_c''$ and $B_d=Y_c'-Y_c''$
- We suppose that (1,3) and (2,4) are uncoupled; $s_{24}=s_{13}=0$
- We suppose that all ports are matched: $s_{ii}=0$.

Without showing every eigencircuit, the eigenvalues are obtained:

$$Y_{\lambda 1}=j(Y_c'+Y_c'')\Rightarrow S_{\lambda 1}=\frac{Y_0-jB_s}{Y_0+jB_s} \qquad Y_{\lambda 2}=-j(Y_c'+Y_c'')\Rightarrow S_{\lambda 2}=\frac{Y_0+jB_d}{Y_0-jB_d}$$

$$Y_{\lambda 3}=j(Y_c'-Y_c'')\Rightarrow S_{\lambda 3}=\frac{Y_0-jB_d}{Y_0+jB_d} \qquad Y_{\lambda 4}=-j(Y_c'+Y_c'')\Rightarrow S_{\lambda 4}=\frac{Y_0+jB_s}{Y_0-jB_s}$$

Imposing the conditions of matchig and un-coupling:

$$\begin{cases}s_{11}=0\\s_{13}=0\end{cases} \Rightarrow \begin{cases}S_{\lambda 1}+S_{\lambda 2}=0\\S_{\lambda 3}+S_{\lambda 4}=0\end{cases} \Rightarrow \frac{B_sB_d}{Y_0^2}=1\rightarrow Y_c'^2-Y_c''^2=Y_0^2$$

Defining $b_s=\dfrac{B_s}{Y_0}$:

$$s_{12}=\frac{1}{4}(S_{\lambda 1}-S_{\lambda 2}+S_{\lambda 3}-S_{\lambda 4})=\frac{1}{2}(S_{\lambda 1}-S_{\lambda 4})=\frac{-2b_sj}{1+b_s^2}$$

And $s_{14}=\dfrac{1-b_s^2}{1+b_s^2}$.

From the unitary condition: $\phi_{12} - \phi_{14} = \pm\pi/2$. If $\phi_{12} = -\pi/2 \Rightarrow \phi_{14} = \pi$. This means that b_s must be greater than 1, otherwise s_{14} is not negative (this is logical assuming positive the input at port 1 and considering a change oh phase by 180°).
Imposing $|s_{14}|^2 = C$, we obtain b_s:

$$s_{14} = \frac{b_s^2 - 1}{b_s^2 + 1} \Rightarrow b_s = \sqrt{\frac{1+\sqrt{C}}{1-\sqrt{C}}} = \frac{Y_c' + Y_c''}{Y_0}$$

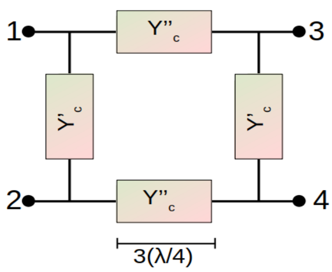
And finally:

$$Y_c' = Y_0 \frac{1}{\sqrt{1-C}} \qquad\qquad Y_c'' = Y_0 \sqrt{\frac{C}{1-C}}$$

More values: $s_{14} = s_{23} = -\sqrt{C}$ and $s_{12} = s_{34} = -j\sqrt{1-C}$.

2. Rat Race

- $s_{11} = s_{22} = s_{33} = s_{44} = 0$
- $s_{14} = s_{23} = 0$
- $|s_{13}|^2 = |s_{24}|^2 = C$
- $|s_{12}|^2 = |s_{21}|^2 = 1 - C$

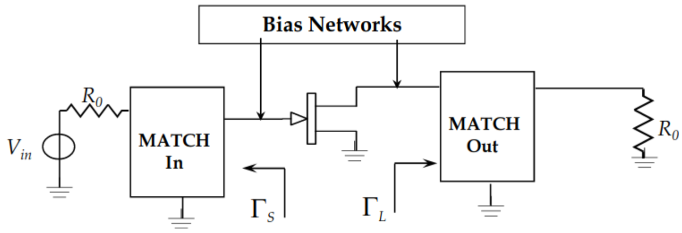


Design equations: $Y_c' = Y_o\sqrt{1-C}$ $\qquad Y_c'' = Y_0\sqrt{C}$

Scattering parameters: $s_{13} = -j\sqrt{C}$ $\qquad s_{24} = j\sqrt{C}$ $\qquad s_{12} = -j\sqrt{1-C}$

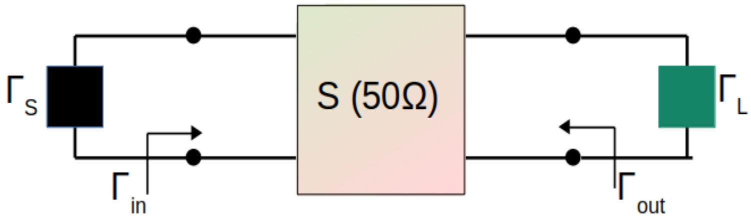
For $C = 0.5(3dB)$: $Z_c' = \dfrac{Z_0}{\sqrt{1-C}} = \dfrac{Z_0}{\sqrt{0.5}}$ $\qquad Z_c'' = \dfrac{Z_0}{\sqrt{C}} = \dfrac{Z_0}{\sqrt{0.5}}$

10 Microwave Amplifiers



Since these devices must be biased, the biasing must be separated by the RF system, thus we need decoupling networs. In case of small signal operations, we can modelize them with a suited scattering matrix.

10.1 Active Device Representation



$$\Gamma_S = \frac{Z_S - 50}{Z_S + 50}$$

$$\Gamma_L = \frac{Z_L - 50}{Z_L + 50}$$

Using suitable formulas, is possible to compute the transducer gain and the reflection coefficients at the input and output of the device.

$$\Gamma_{in} = \frac{Z_{in} - 50}{Z_{in} + 50} = s_{11} + \frac{\Gamma_L s_{12} s_{21}}{(1 - \Gamma_L s_{22})} \qquad \Gamma_{out} = \frac{Z_{out} - 50}{Z_{out} + 50} = s_{22} + \frac{\Gamma_S s_{12} s_{21}}{(1 - \Gamma_S s_{11})}$$

$$G_T = |s_{21}|^2 \frac{\left(1 - |\Gamma_L|^2\right) \cdot \left(1 - |\Gamma_L|^2\right)}{|(1 - \Gamma_S \cdot s_{11})(1 - \Gamma_L \cdot s_{22}) - \Gamma_S \Gamma_L s_{12} s_{21}|^2}$$

A 2-port network operating as an amplifier must be stable.
A network is unconditionally stable provided that **both** these following equations are verified for whatever value of Γ_L and Γ_S:

$$\begin{cases} |\Gamma_{in}| < 1 \\ |\Gamma_{out}| < 1 \end{cases} \longleftrightarrow k = \frac{1 - |s_{11}|^2 - |s_{22}|^2 - |s_{11} \cdot s_{22} - s_{12} \cdot s_{21}|^2}{2|s_{12} s_{21}|} > 1 \quad \wedge \quad \det[\bar{\bar{S}}] < 1$$

Once these conditions are satisfied, we can calculated a pair of optimum values, $\Gamma_{S,opt}, \Gamma_{L,opt}$ for which the gain is maximum and the conjugate matching is achieved:

$$G_{T,max} = \left|\frac{s_{21}}{s_{21}}\right| (k - \sqrt{k^2 - 1})$$

As for conjugate matching:

$$\begin{cases} \Gamma_{S,opt} = \Gamma_{in}^* \\ \Gamma_{L,opt} = \Gamma_{out}^* \end{cases} \longrightarrow \begin{cases} \Gamma_{S,opt} = \dfrac{C_G^* - [B_g - (B_g^2 - 4|C_g|^2)^{1/2}]}{2|C_g|^2} \\ \\ \Gamma_{L,opt} = \dfrac{C_L^* - [B_L - (B_L^2 - 4|C_L|^2)^{1/2}]}{2|C_L|^2} \end{cases}$$

Where:

$$B_g = 1 + |s_{11}|^2 - |s_{22}|^2 - |s_{11}s_{22} - s_{12}s_{21}|^2 \qquad B_L = 1 - |s_{11}|^2 - |s_{22}|^2 - |s_{11}s_{22} - s_{12}s_{21}|^2$$

$$C_g = s_{11} - (s_{11}s_{22} - s_{12}s_{21})s_{22}^* \qquad C_L = s_{22} - (s_{11}s_{22} - s_{12}s_{21})s_{11}^*$$

NOTE THAT ABOVE EQUATIONS HOLD FOR $s_{12} \neq 0$.

10.2 Potentially unstable devices

If $k < 1$ we cannot say that the device is unconditionally stable. Hence, does not exist a pair of optima values of Γ for which G_T is maximum. As a matter of fact, *in case of instability* G_T si infinite.

To find admissible values of Γ we need to come back to the conditions introduced in the previous chapter:

$$\begin{cases} |\Gamma_{in}| = \left| s_{11} + \dfrac{\Gamma_L s_{12}s_{21}}{(1-\Gamma_L s_{22})} \right| < 1 \\ \\ |\Gamma_{out}| = \left| s_{22} + \dfrac{\Gamma_S s_{12}s_{21}}{(1-\Gamma_S s_{11})} \right| < 1 \end{cases}$$

While for what concerns the transducer gain we can consider the reference value of $G_{T,max} = \left|\dfrac{s_{21}}{s_{12}}\right|$.

<u>Admissible region Γ_L</u>

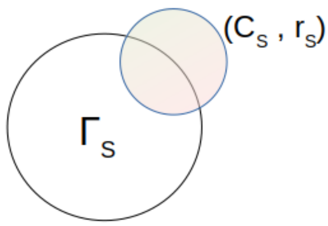
The boundary condition is derived by the following equation:

$$|\Gamma_{out}| = \left| s_{22} + \frac{\Gamma_S s_{12}s_{21}}{(1-\Gamma_S s_{11})} \right| = 1$$

The equation define a circles with the following center and radius:

$$C_S = \frac{s_{22}\Delta^* - s_{11}^*}{|\Delta|^2 - |s_{11}|^2}$$

$$r_S = \frac{|s_12 \cdot s_{21}|}{|\Delta|^2 - |s_{11}|^2}$$



<u>Admissible region Γ_L</u>

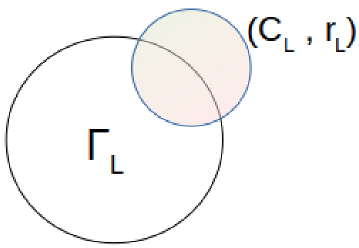
The boundary condition is derived by the following equation:

$$|\Gamma_{in}| = \left| s_{11} + \frac{\Gamma_L s_{12}s_{21}}{(1-\Gamma_L s_{22})} \right| = 1$$

The equation define a circles with the following center and radius:

$$C_L = \frac{s_{11}\Delta^* - s_{22}^*}{|\Delta|^2 - |s_{22}|^2}$$

$$r_L = \frac{|s_12 \cdot s_{21}|}{|\Delta|^2 - |s_{22}|^2}$$



Identification of admissible region

We need to observe the vaues of $\Gamma_{in}, \Gamma_{out}$ when $\Gamma_S = \Gamma_L = 0$. Taking into account that in this case $\Gamma_{in} = s_{11}$ and $\Gamma_{out} = s_{22}$:

- the stable region for $\Gamma_L(\Gamma_S)$ is **outside** the instability circle if:
  - $|s_{11}|(|s_{22}|) < 1$ and the circle does not enclose the center of the Chart
  - $|s_{11}|(|s_{22}|) > 1$ and the circle encloses the center of the chart
- the stable region for $\Gamma_L(\Gamma_S)$ is **inside** the instability circle if:
  - $|s_{11}|(|s_{22}|) > 1$ and the circle does not enclose the cener of the chart
  - $|s_{11}|(|s_{22}|) < 1$ and the circle encloses the center of the chart

Design with potentially unstable devices

There is not a unique solution:

1. : Γ_L is choosen in its stable region and Γ_S is computed in order to achieve maximum G_T. Remember that also Γ_S must result in its stabe region.

$$\Gamma_S = \Gamma_{in}^* = \left(s_{11} + \frac{s_{12}s_{21}\Gamma_L}{1 - s_{22} \cdot \Gamma_L}\right)^*$$

2. : Γ_S is choosen inside its stabel region and Γ_L is computed to achieve maximum G_T:

$$\Gamma_L = \Gamma_{out}^* = \left(s_{22} + \frac{s_{12}s_{21}\Gamma_S}{1 - s_{11} \cdot \Gamma_S}\right)^*$$

From the definition of **power gain** the following expression is derived:

$$G_P = |s_{21}|^2 \frac{(1 - \Gamma_L^2)}{1 - |s_{11}| + |\Gamma_L|^2 \cdot (|s_{22}|^2 - |\Delta|^2) - 2Re[\Gamma_L(s_{22} - \Delta \cdot s_{11}^*)]}$$

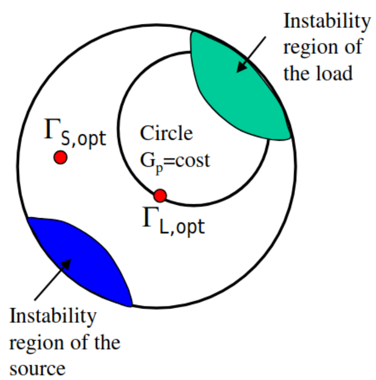
In general $G_P \geq G_T$, it's equal if the input is matched, and it's independent from Γ_S. Moreover, drawing this equation on the plane of Γ_L, we find a curve along which G_P is constant: it's a circle with...

$$Center = C_P = \frac{g_P(s_{22}^* - \Delta^* \cdot s_{11})}{1 + g_P(|s_{22}|^2 - |\Delta|^2)} \qquad radius = r_P = \frac{(1 - 2k|s_{12}s_{21}|g_P + |s_{12}s_{21}|^2 g_P^2)^{1/2}}{1 + g_P(|s_{22}|^2 - |\Delta|^2)}$$

$$g_P = \frac{G_P}{|s_{21}|^2}$$

Now, suppose that G_P is assigned:

- Draw the circle $G_P = G_T$ on the Smith Chart representing Γ_L
- Select $\Gamma_{L,opt}$ on this circle and verify that's inside the stability region of the load
- Compute $\Gamma_{S,opt}$ from the equation for the conjugate matching at the input
- verify that this value is in the stability region of Γ_S.



From the definition of **available gain** the following expression is derived:

$$G_A = |s_{21}|^2 \frac{(1-\Gamma_S^2)}{1-|s_{22}|+|\Gamma_S|^2 \cdot (|s_{11}|^2 - |\Delta|^2) - 2Re[\Gamma_S(s_{11} - \Delta \cdot s_{22}^*)]}$$

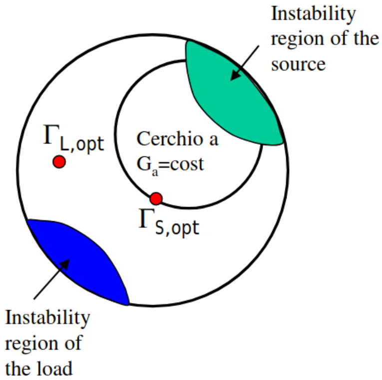
In general $G_A \geq G_T$, it's equal if the output is matched, and it's independent from Γ_S. Moreover, drawing this equation on the plane of Γ_S, we find a curve along which G_A is constant: it's a circle with...

$$Center = C_P = \frac{g_A(s_{11}^* - \Delta^* \cdot s_{22})}{1+g_A(|s_{11}|^2 - |\Delta|^2)} \qquad radius = r_A = \frac{(1-2k|s_{12}s_{21}|g_A + |s_{12}s_{21}|^2 g_A^2)^{1/2}}{1+g_A(|s_{22}|^2 - |\Delta|^2)}$$

$$g_A = \frac{G_A}{|s_{21}|^2}$$

Now, suppose that G_A is assigned:

- Draw the circle $G_A = G_T$ on the Smith Chart representing Γ_S
- Select $\Gamma_{S,opt}$ on this circle and verify that's inside the stability region of the source
- Compute $\Gamma_{L,opt}$ from the equation for the conjugate matching at the output
- verify that this value is in the stability region of Γ_L.



Design Result

Case 1	Case 2
Trasducer gain imposed Input matched (NOT the output)	Trasduced gain imposed Output matched (NOT the input)

If the network is lossless, also the input or output of the amplifier is matched!

10.3 Main sources ot electric noise

Thermal Noise, caused by dissipation:	η = Power spectral density = $K \cdot T$
Shot Noise (discrete nature of junctions current):	η = Power spectral density = $2q \cdot I$
Flicker Noise (defects of crystals structures):	$G(f)$ = Power Spectrum = $\bar{K}\frac{I^a}{f^b} \quad [W/Hz]$
Added power from the 2-port:	N_{DB}
Total Noise Power:	$P_{N,out} = P_{N,in} G_A + N_{DB}$
Noise Figure (function of frequency and Γ_S):	$NF = \frac{P_{N,out}}{G_A P_{N,in}}$
Noise dependance on Γ_S:	$NF = (NF)_{min} + 4r_n \frac{\|\Gamma_S - \Gamma_{min}\|^2}{\|1+\Gamma_{min}\|^2 (1-\|\Gamma_S\|^2)}$

As for the latter definition, all parameters depend on frequency and "r_n" is known as normalized noise resistance.

If we plot the NF dependinf on Γ_S on the Smith Chart we will find a circle with the following parameters:

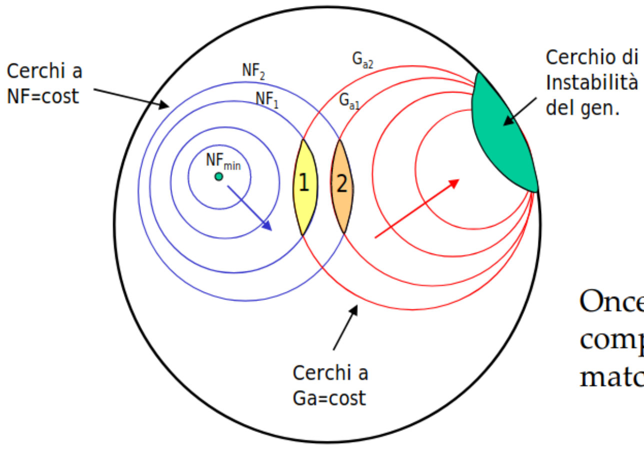
$$C_F = \frac{\Gamma_{min}}{1+N_i} \qquad\qquad r_F = \frac{1}{1+N_i}\sqrt{N_i^2 + N_i(1-|\Gamma_{min}|^2)}$$

$$N_i = \frac{NF-(NF)_{min}}{4r_n}(1+|\Gamma_{min}|^2)$$

Noise Figure for cascaded stages: $NF_{tot} = NF_1 + \frac{NF_2 - 1}{G_{a1}} + \frac{NF_3 - 1}{G_{a2}} + \cdots$

<u>Design of a Low Noise Amplifier</u>

In general the value of Γ_S that determines the minimum value of NF is not the same that maximize G_T. The choice of Γ_S is then the result of a compromise.
If we plot the circles defined by $NF = const.$ and $G_A = const.$ on the Smith Chart, we would see that some pairs of circles share a common areas, resulting form the intersection. We choose Γ_S within this area.



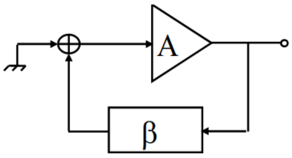
In zone **1:** NF≤ NF_1 and $G_a \geq G_{a2}$
NF is previliged

In zone **2:** NF≤ NF_2 and $G_a \geq G_{a1}$
G_a is previliged

Once assigned $\Gamma_{s,opt}$, $\Gamma_{L,opt}$ is computed by imposing the matching at output (then G_T=G_a)

11 Oscillators

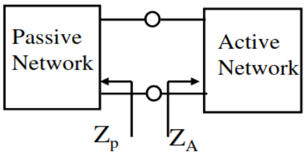
11.1 Classification and main parameters



Feedback Oscillators

$$G_{LOOP} = A \cdot \beta(j\omega_0) = 1 \rightarrow |G_{LOOP}(j\omega_0)| = 1, \angle(G_{LOOP}(j\omega_0)) = 2n \cdot \pi$$

Negative resistance oscillators

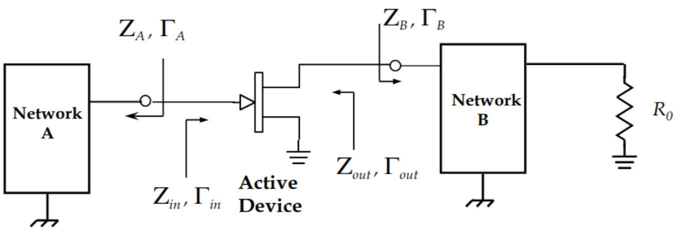


$$R_A = R_B, \quad X_A(\omega_0) + X_B(\omega_0) = 0$$

- Indirect stability coefficient: $S_{F,\Phi} = \omega_0 \left| \frac{d\Phi_{LOOP}}{d\omega} \right|_{\omega=\omega_0}$ $\quad S_{F,X} = \omega_0 \left| \frac{d[X_A(\omega) + X_B(\omega)]}{d\omega} \right|_{\omega=\omega_0}$
- Harmonic distortion: It defines numerically the amplitude of the harmonic referred to the fundamental ω_0
- Phase and Amplitude Noise: random fluctuations of amplitude and phase, the second type is unavoidable

$$\omega_0' = \omega_0 \left(1 + \frac{\Delta\Phi}{S_{F,\Phi}}\right) \longleftrightarrow \frac{\omega_0' - \omega_0}{\omega_0} = \frac{\Delta\Phi}{S_{F,\Phi}}$$

11.2 Configuration and conditions

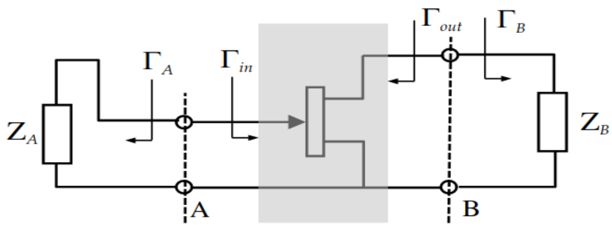


In this case we need negative resistances at input and output in order to have a sinusoidal signal. This means that input/output reflection coefficient are larger than 1: the device must be potentially unstable.

$$|\Gamma_{in}| > 1 \wedge |\Gamma_{out}| > 1$$

To increment the instability of the device we must decrease K. We achieve this goal by changing the reference terminal of the active device or by introducing a positive feedback.

For what concerns the actual conditions:



Steady-state conditions: it's sufficient that only one of the following is satisfied

$$\Gamma_{in}(j\omega_0) \cdot \Gamma_A(j\omega_0) = 1 \quad \Rightarrow \quad (Z_{in} + Z_A) = 0, \quad (Y_{in} + Y_A) = 0$$

$$\Gamma_{out}(j\omega_0) \cdot \Gamma_B(j\omega_0) = 1 \quad \Rightarrow \quad (Z_{out} + Z_B) = 0, \quad (Y_{out} + Y_B) = 0$$

Start-up conditions: both the following equations must be satisfied

$$|\Gamma_{in}(j\omega_0) \cdot \Gamma_A(j\omega_0)| > 1 \quad \Rightarrow \quad Re(Z_{in} + Z_A) < 0, \quad Re(Y_{in} + Y_A) < 0$$

$$|\Gamma_{out}(j\omega_0) \cdot \Gamma_B(j\omega_0)| > 1 \quad \Rightarrow \quad Re(Z_{out} + Z_B) < 0, \quad Re(Y_{out} + Y_B) < 0$$

Note that we have these relations because the poles must be on the right-hand plane. Anyway, after the start the oscillation grows and the poles move towards the left-hand plane. Once they have reached the imaginary axis, the oscillation remains with constant amplitude and the transient is concluded.

The frequency of the oscillation is derived by the regime condition combined with the start-up requirements:

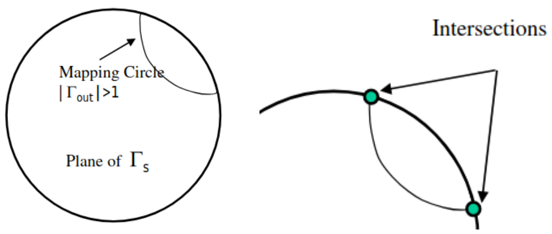
$\lvert\Gamma_{in}(j\omega_0) \cdot \Gamma_A(j\omega_0)\rvert > 1, \quad \lvert\Gamma_{out}(j\omega_0) \cdot \Gamma_B(j\omega_0)\rvert > 1$	$R_{in}(j\omega_0) + R_A(j\omega_0) < 0, \quad R_{out}(j\omega_0) + R_B(j\omega_0) < 0$
$\angle(\Gamma_{in}(j\omega_0) \cdot \Gamma_A(j\omega_0)) = 0$	$X_{in}(j\omega_0) + X_A(j\omega_0) = 0$

To increase the stability only the phase of Γ_A should determine ω_0. Note that once ω_0 is imposed at section A, it is also verified (at regime) at section B.

11.3 General Design procedure

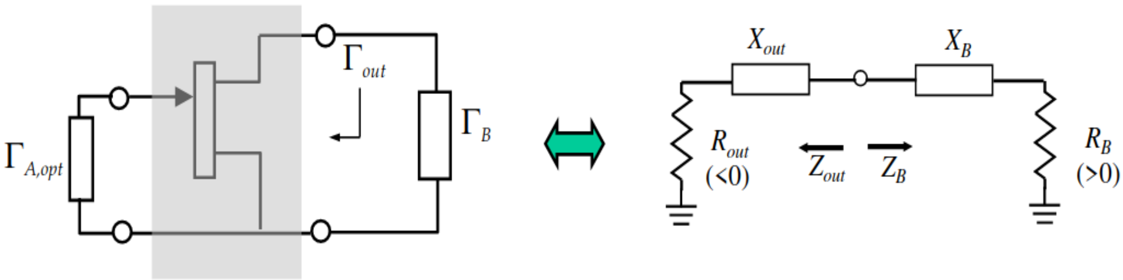
We look for the values of Γ_A and Γ_B that allow the start of oscillation. We fix $|\Gamma_A| = 1$, that is, the network A is made of reactive components. The unknown paramteres are: $\angle\Gamma_A$, $|\Gamma_B|$ and $\angle\Gamma_B$.

1. Evaluation of $\angle\Gamma_A$.
   - The value of $|\Gamma_{out}| > 1$ is assigned
   - The corresponding circle representing Γ_{out} is drawn on the plane of Γ_S
   - Look for intersections of the circles (where $|\Gamma_S| = 1$). Otherwise re-assign Γ_{out}

   One of the points of intersection is selected: $\Gamma_S = \Gamma_{A,opt}$ and thus the network A is sythesized as a reactive 1-port network.

2. Evaluation of Γ_B.
   - $\Gamma_{out} = s_{11} + \dfrac{s_{12}s_{21}\Gamma_{A,opt}}{1 - s_{22}\Gamma_{A,opt}}$
   - Z_{out} or Y_{out} is derived from Γ_{out}, and it has a negative real part
   - Hence, imposing the start of oscillation: $Z_{B,opt} = {}^{R_{out}}/_3 - jX_{out}$ or $Y_{B,opt} = {}^{G_{out}}/_3 - jB_{out}$



- Z_{in} or Y_{in} are derived from $Z_{B,opt}$ and $Y_{B,opt}$
- One of the following conditions must be verified, otherwise re-assign $Z_{B,opt}$ or $|\Gamma_{out}|$ and repeat the procedure:

  $$(R_{in} - R_{A,opt}) < 0 \qquad\qquad (G_{in} + G_{A,opt}) < 0$$

  Note that the first equation is equivalent to $|\Gamma_{in} \cdot \Gamma_{A,opt}| > 1$
- In the end $\Gamma_{B,opt}$ is evaluated from $Z_{B,opt}$ and the network B is synthesized by imposing impedance trasformation of the load

11.4 Noise in oscillators

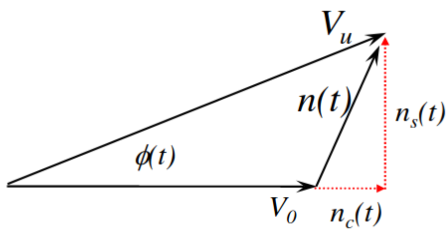
The noise in the electric circuits determines fluctations of the instantaneous phase of the generated signal. These fluctuations can be seen as a modulation of the sinusoidal proceduced by noise

Representation in the time domain:	$V(t) = V_s \cos(\omega t)$
Relation frequency-phase:	$f = \frac{1}{2\pi} \cdot \frac{d\Phi}{dt}$
Power spectrum of phase fluctuations:	$\Delta\Phi^2$
Small-signal modulation:	$\Delta\Phi \ll 1$
Power density:	$L(f)$

For small modulation angle L(f) has the same shape of $\Delta\Phi^2$.

Phasorial description:

$n(t)$ is the noise phasor, with random magnitude and phase. $n(t)$ can be divided in components: "in phase" $n_c(t)$ and "in quadrature" $n_s(t)$.
Half of the overall power density is associated to each of them.



Suppose that $n_s(t) \ll V_u$ and so $V_u \approx V_0$:

$$\Phi(t) = \sin^{-1}\left(\frac{n_s(t)}{V_u}\right) \cong \frac{n_s(t)}{V_u} \cong \frac{n_s(t)}{V_0}$$

The spectrum of phase fluctuations is proportional to the spectrum of added noise:

$$S_\Phi(f) = \frac{N(f)}{2P_0}$$

Whenever we have to work with a positive feedback network, the noise frequency components close to the oscillation frequency are amplified byt he loop. Ultimately a broadening of the ideal spectral line is produced.

For the phase noise we can trust the Leeson's model, whose relation put in relations the power spectrum of the phase fluctuation, the noise spectrum and the indirect stability of the oscillator:

$$S_\Phi(f) \propto \left(\frac{f_0}{S_F}\right)^2 \frac{N(f_\Delta)}{2(f_\Delta)^2}$$

The parameter f_Δ represents the deviation with respect the oscillation frequency f_0. This model is accurate for small values of f_Δ.
Actually, the Leeson's model holds true for noise components inside the band of G_loop. The band B is mainly determined by the selectiity of the feedback network:

$$B = \frac{f_0}{Q_0}$$

For what concerns the specturm of $V_u(f)$: around f_0 is proportional to $S_\Phi(f_\Delta)$ and is symmetric only if phase noise is present.
The principal figure of merit is the CNR, or carrier to noise ratio:

$$CNR(f_\Delta) = \frac{P_0}{S_{V_0}(f_0 + f_\Delta)} = \frac{P_0}{S_\Phi(f)}$$

12 Mixers

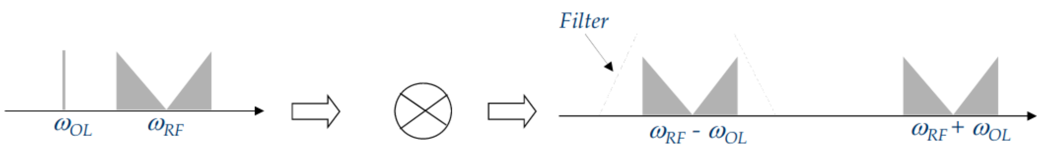
12.1 "Basics"

We need mixers to traslate the frequency of the RF signals. It implies necesserily a multiplication:

$$V_U = V_{RF} \cdot V_{OL} = V_M(t)\cos(\omega_{RF}t + \Phi(t)) \cdot V_0\cos(\omega_{OL}t) = V_M(t)V_0\cos(\omega_{RF}t + \Phi(t))\cos(\omega_{OL}t)$$

Hence we obtain the following expression:

$$V_U = \frac{V_0}{2}[V_M(t)\cos((\omega_{RF} - \omega_{OL})t + \Phi(t)) + V_M\cos((\omega_{RF} + \omega_{OL})t + \Phi(t))]$$



Since the traslation produces two components, we need a filter in the communication filter the select the right one.

12.2 Practical implementation and Classes

At mirowave frequencies it's much easier to realize the frequency traslsation exploiting the 2-port non-linear devices:

$$V_{out} = a_1V_{in} + a_2V_{in}^2 + a_3V_{in}^3 + \cdots$$

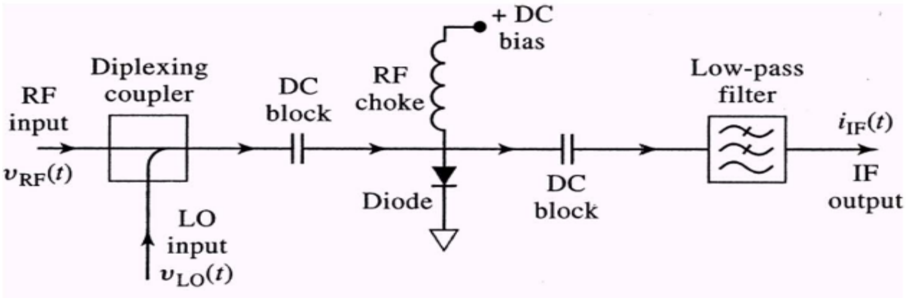
Since $V_{in} = V_{RF} + V_{OL}$:

$$V_{out} = \frac{a_2}{2}(V_M + V_0) + a_1(V_0\cos(\omega_{OL})t + V_M\cos(\omega_{RF}t)) +$$

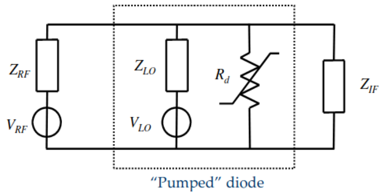
$$+\frac{a_2}{2}(V_0^2\cos(2\omega_0t) + V_M^2\cos(2\omega_{RF}t)) + a_2(2V_0V_M\cos(\omega_0t)\cdot\cos(\omega_{RF})) + \ldots$$

The most used non-linear device is the Shotcky diode. There are two main classes of microwave mixers: Mixers with a single diode and balanced mixers (2 or 4 diodes).

<u>Single-ended mixers</u>



Equivalent circuit:



The diode R_d is characterized by the following function: $I_D = I_s \left(e^{\frac{V_D}{V_T}} - 1 \right)$. It can be assumed memory-less.

Effects of distortion:

- The spectrum of the frequency-traslated signal around the new carrier frequency is different from the original one
- New replicas of the original RF signal, LO and combination of them are generated at different carrier frequencies

The local linearity is described with the same parameters seen for the amplifier (P_{1dB}, IP_3).